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Economic Dynamics Model Using q-Laplace's Transform

Pintu Bhattacharya[1], Dharam Prakash Paswan[2], Kundan Kumar[3]
[1]Assistant professor, Dept. of Physics, L. N. College, B. R. A. B. University, Muzaffarpur, Bihar, India.
[2]Assistant professor, Dept. of Economics, L. N. College, B. R. A. B. University, Muzaffarpur, Bihar, India.
[3]Research Scholar, Dept. of Economics, B. R. A. B. University, Muzaffarpur, Bihar, India
Emails: *pintu@lncollege.ac.in[1], kdharam7979@gmail.com[2], kkundan.lnc@gmail.com[3]*

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Abstract
Here, for the first time, we introduce the quantum Laplace transform as a tool to analyze dynamic models in economic theory. In order to do it, at first, we revisit some useful essential preliminaries of q-calculus as well as q-Laplace transform and then apply this framework to three important cases: the Walrasian price stability model, the measurement of input response in an inflationary setting, and the model of farm sponsored job training. The solutions derived through this approach are studied numerically that shows this method provides a clear and systematic way to interpret dynamic behavior in economics. A noteworthy feature of this framework is its consistency, and it is notable that as the parameter $q \to 1$, all the outcomes reduce smoothly to those obtained via the classical Laplace transform. This convergence highlights the robustness of the method that opens a promising pathway for deploying q-calculus in economic dynamics. In short, this approach opens new research opportunities for both theoretical and applied economics..

1. Introduction

Quantum calculus is a well-known field of mathematics that excludes any concept of limit within the definition of q-derivative [1-3]. Recently, a lot of research has been published to study physical as well as mathematical findings by employing different methods of q-calculus [4-12]. In contrast, economic dynamics studies the evolution of economic variables over time and often modelled through systems of difference or differential equations. Occasionally, the analysis of such models relies on classical methods, in which it does not yield closed-form solutions and it requires numerical approximations under restrictive assumptions [12-15]. In order to address this challenge, this paper explores the application of q-integral transform methods [16-18], particularly the Laplace' transforms in analysing selected models which are more familiar in economic theory. This technique is well established in engineering and signal processing through concepts such as convolution, transfer functions, and the Heaviside step function, but it remains underutilized in economics till today. In order to tackle this problem, at first, we investigate the feasibility of employing such tools in economic dynamics [13-15]. After that, we focus on both the Walrasian price stability model and the measurement of input response in an inflationary model, reformulated within the framework of q-calculus. By extending integral transform techniques to the q-calculus setting for the first time, we demonstrate how closed-form solutions can be derived more effectively that provides a novel analytical pathway for handling complex dynamic systems in economics. The findings indicate that integral transforms, when adapted through q-analysis, not only enrich the methodological toolkit of economic theory but also open avenues for future research in nonlinear and time-dependent economic models.

1.1. Essential Preliminaries of quantum calculus

Here in this section, we preset all the essential basics of q-calculus as described in ref. [1-5]. First, we write basic definitions of q-algebra and then we display all the useful formula of q-Laplace's transform. The q −number (l) , q −factorial, small q −exponential of kt, q −binomial function, and q-derivative of a function $f(x)$ as follows

$$[n]_q = \frac{q^n - 1}{q-1}, q \in \mathbb{C} - \{-1\}; n \in \mathbb{C}. \quad \dots (1)$$

$$[l]_q! = \frac{[0]_q! = 1}{\prod_{l=1}^{l} [l]_q}, \quad \dots (2)$$

When, $q \neq 1, l \in \mathbb{N}, 0 \leq q \leq 1$

$$e_q(kt) = \sum_{n=0}^{\infty} \frac{(kt)^n}{[n]_q!}, 0 < |q| < 1 \quad \dots (3)$$

$$e_q(y_1).e_q(y_2) = e_q(y_1 + y_2), \quad \dots (4)$$

when $y_1 y_2 = q y_2 y_1$.

$$\binom{m}{n}_q = \frac{[m]_q!}{[m]_q! [m-n]_q!} \quad \dots (5)$$

when, $m = 0,1,2, \dots n$.

$$D_q f(t) = \frac{d_q f(t)}{d_q t} = \frac{f(qt) - f(t)}{(q-1)t}, \quad \dots (6)$$

For, $0 < |q| < 1$.

1.2 Basics of q-Laplace' Transform

Laplace transform of a function $f(t)$ is given by

$$\mathcal{F}(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt \quad \dots (7)$$

But, if $f(t)$ represents a function of discrete variable, then we can apply Z-transform as,

$$\mathcal{F}(t) = Z\{f(t)\} = \sum_{j=0}^{\infty} f(j) t^{-j}, t = e^{-s} \quad \dots (8)$$

However, Laplace transform in q-calculus can be described in different ways [16-18]. In our study, we shall use the procedure as described in [16] for its usefulness. Thus, q-Laplace transform can be defined as,

$$\mathcal{F}_q(s) = \mathcal{L}_q\{f(t)\} = \int_0^{\infty} e_{\frac{1}{q}}^{-st} f(t) d_q t, \quad \dots (9)$$

$s = g + ih \in C$. Here, $f(t)$ is known as the q-original of $\mathcal{F}_q(s)$. And $\mathcal{F}_q(s)$ is basically the q −image of $f(t)$ as described by q −Laplace' transform. Therefore, $\mathcal{F}_q(s)$ is basically the q −image of $f(t)$ as described by q −Laplace' transform. Therefore, we can say clearly,

$$\mathcal{L}_q\{\alpha f_1(t) + \beta f_2(t)\} = \mathcal{L}_q\{\alpha f_1\} + \mathcal{L}_q\{\beta f_2(t)\} \quad \dots (10)$$

Where, α and β are two constants. From (9), q-Laplace transform of any arbitrary constant c is

$$\mathcal{F}_q(s) = \mathcal{L}_q\{c\} = \int_0^{\infty} e_{\frac{1}{q}}^{-st} c d_q t$$

Which gives,

$$\mathcal{F}_q(s) = -\frac{qc}{s} \int_0^{\infty} D_q e_{\frac{1}{q}}^{-\frac{s}{q}t} d_q t = q\left(\frac{c}{s}\right) \quad \dots (11)$$

Additionally, the q-Laplace transformation of derivative is,

$$\mathcal{G}_q(s) = \mathcal{L}_q\{D_q f(t)\} = \int_0^{\infty} e_{\frac{1}{q}}^{-st} \{D_q f(t)\} d_q t \quad \dots (12)$$

Using the q-integration of by-parts and rearranging, we obtain,

$$\mathcal{G}_q(s) = \left[f(t) e_{\frac{1}{q}}^{-st}\right]_0^{\infty} + s\int_0^{\infty} f(qt) e_{\frac{1}{q}}^{-sqt} d_q t \quad \dots (13)$$

And then,

$$\mathcal{G}_q(s) = -f(0) + \frac{s}{q}\int_0^{\infty} f(t) e_{\frac{1}{q}}^{-st} d_q t \quad \dots (13.a)$$

$$\mathcal{G}_q(s) = -f(0) + \frac{s}{q} F_q(s) \quad \dots (13.b)$$

Setting, $s' = s/q$ for simplicity, we can write (13.b) as,

$$\mathcal{G}_q(s) = s' \mathcal{F}_q(s) - f(0) \quad \dots (14)$$

Suppose a function

$$\theta(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases}$$

... (15)

Now,

$$\mathcal{L}_q\{\theta(t)\} = \int_0^{\infty} \theta(t) e_{\frac{1}{q}}^{-st}\, d_q t$$

... (15.a)

As per the definition of $\theta(t)$ as presented in (15), (15.a) becomes,

$$\mathcal{L}_q\{\theta(t)\} = \int_0^{\infty} e_{\frac{1}{q}}^{-st}\, d_q t = -\frac{q}{s}\int_0^{\infty} D_q e_{\frac{1}{q}}^{-\frac{s}{q}t}\, d_q t$$

Or,

$$\mathcal{L}_q\{\theta(t)\} = \left(\frac{q}{s}\right)$$

... (15.b)

Then, we can write $f(t) = f(t)\theta(t)$ for $t \geq 0$. Hence,

$$\mathcal{L}_q\{f\ (t-a)\} = \int_a^{\infty} e_{\frac{1}{q}}^{-st} f\ (t-a)\theta\ (t-a) d_q t.$$

... (15.c)

Now, setting $t - a = t$, we can write,

$$\mathcal{L}_q\{f\ (t-a)\} = \int_0^{\infty} \frac{t+a}{t} e_q^{-st} e_{\frac{1}{q}}^{-s(t+a)} f\ (t) d_q t.$$

$$= e_{\frac{1}{q}}^{-sa} \mathcal{L}_q \left\{\frac{t+a}{t} e_q^{-sa} e_q^{-st} e_{\frac{1}{q}}^{-s(t+a)} f\ (t)\right\}$$

... (16)

The above expression is not simply understandable for unit heavy-side function. Therefore, to avoid cumbersome process, we may consider a new type q-Laplace's transform of a unit heavy-side step function (see ref. 16) as,

$$\vartheta(t-a) = \begin{cases} 0, & 0 \leq t < a \\ 1, & t \geq 0 \end{cases}$$

... (17)

And the q-Laplace transform of $\vartheta(t-a)$ is given by

$$\mathcal{L}_q\{\vartheta(t-a)\} = \frac{1}{s}\left(e_{\frac{1}{q}}^{-as}\right)$$

... (17.a)

But fortunately, from (16), $\mathcal{L}_q\{\theta(t)\}$ will be $1/s$ if and only if $f(t) = e_q^{-st}$. But, if consider the new type of q-Laplace transform, then the result will be same as in the form in conventional one except the exponentials will be q-exponentials in the q-framework. And the product of transform is defined as,

$$f(t) * g(t) = \mathcal{L}_q\{f(t)\}\mathcal{L}_q\{f(t)\} = \mathcal{F}_q(s)\, \mathcal{G}_q(s)$$

... (18)

And, q –Laplace transform of $f(x) = \sum_{n=0}^{\infty} k_n x^n$ is,

$$\mathcal{F}_q(s) = \sum_{n=0}^{\infty} k_n\, \mathcal{L}_q\{t^n\} = \frac{q}{s}\sum_{n=0}^{\infty} k_n [n]_q! \left(\frac{q}{s}\right)^n$$

... (19)

Thus, q –Laplace transform of $f(t) = Ae_q^{kt}$ is,

$$\mathcal{F}_q(s) = L_q\{Ae_q^{kt}\} = \sum_{n=0}^{\infty} \frac{Ak^n}{[n]_q!} L_q\{t^n\} = A\frac{q}{s-qk}$$

Note that, in most of the cases, the q-original of a given q-image can be obtained by using the results of the transformation of basic elementary functions combined with the application of the properties of the q-Laplace transform [16,17], and we may write,

$$L_q^{-1}\left\{A\frac{q}{s-qk}\right\} = L_q^{-1}\left\{\frac{A}{\left({}^{s}/_{q} - k\right)}\right\}$$

$$= \frac{Aq}{q} e_q^{kxq/q} = Ae_q^{kx}\ .$$

... (20)

2. Economic Dynamic Model Applying Quantum Laplace' Transform

Economic dynamics explains traditional economic analysis by considering all the key variables evolve over time [12-13]. This temporal perspective is foundational to several advanced domains, such as the study of economic growth, dynamic stochastic general equilibrium models, financial market analysis, and nonlinear economic processes [12-15]. The mathematical framework for all these models is constructed by the conception of discrete-time setting which are naturally modeled with difference equations; whereas continuous-time analysis employs differential equations. For instance, the Walrasian price adjustment mechanism is classically represented by a first-order, non-homogeneous differential equation. The modeling toolkit has further expanded to include integral equations, as explored in contemporary research as described in ref. [14].

2.1 Essential Preliminaries of quantum calculus

The Walrasian price stability model is a classic illustration in microeconomic theory that discuss about the operation of economic dynamics. It explains the process by which market prices adjust over time in response to imbalances between demand and supply. Let us consider a market where the demand and supply for a commodity at any time t can be expressed as,

$$\delta(t) = a_1 - a_2\rho(t),$$

... (21)

and

$$\sigma(t) = a_3 + a_4\rho(t).$$

... (22)

where $\delta(t)$, $\sigma(t)$, $\rho(t)$ represent the quantity demanded, quantity supplied and the price at time t respectively. $a_1, a_2, a_3,$ and are a_4 real-valued parameters. Note that, economically, the price adjustments are driven by disequilibrium which means larger gaps between demand and supply produce faster price movements. Therefore, the time derivative of price is a positive multiple of the difference between demand and supply. Thus,

$$\dot{\rho}(t) = \lambda[\delta(t) - \sigma(t)].$$

... (23)

In (23), $\lambda > 0$ is an arbitrary constant of variation. After substituting $\delta(t)$, and $\sigma(t)$ from (21) & (22), (23) becomes,

$$\dot{\rho}(t) = \lambda[\{a_1 - a_2\rho(t)\} - \{a_3 + a_4\rho(t)\}].$$

... (24)

After rearranging (24), we obtain, the form of as get,

$$\dot{\rho}(t) + \lambda(a_2 + a_4)\rho(t) = \lambda(a_1 - a_3).$$

... (25)

Applying q-Laplace' transform on both side of the differential equation (25), we dynamically stable in this model. Notably, when the q-Laplace transform is applied to this model, the outcome aligns with the results derived through the standard Laplace transform and other conventional solution methods, which demonstrates that the equilibrium dynamics remain reliable within the q-calculus framework.

$$\mathcal{L}_q\{\dot{\rho}(t)\} + \lambda(a_2 + a_4)\mathcal{L}_q\{\rho(t)\} = \mathcal{L}_q\{\lambda(a_1 - a_3)\}.$$

Now using (11), & (13.b), we can write,

$$s'\mathcal{F}_q(s) - \rho(0) + \lambda(a_2 + a_4)\mathcal{F}_q(s) = \frac{(a_1 - a_3)}{s'}$$

Hence,

$$\mathcal{F}_q(s) = \frac{1}{s'}\left[\frac{\lambda(a_1 - a_3) + s'\rho(0)}{s' + \lambda(a_2 + a_4)}\right].$$

... (26)

By partial fraction decomposition,

$$\mathcal{F}_q(s) = \frac{a_1 - a_3}{a_2 + a_4}\left(\frac{1}{s'}\right) + \left[\frac{\rho(0) - \dfrac{a_1 - a_3}{a_2 + a_4}}{s' + \lambda(a_2 + a_4)}\right]$$

... (27)

Considering (27), remembering (20), the time path of the price $[\rho(t)]$variable in the Walrasian price stability model will takes the form as,

$$\rho(t) = \rho_e + [\rho(0) - \rho_e]e_q(-\lambda' t).$$

... (28)

Here, we set $(a_1 - a_3)/(a_2 + a_4) = \rho_e$, and $\lambda(a_2 + a_4) = \lambda'$. From (24), it is also obvious that, after a long-time span (i.e. when $t \to \infty$, $\rho(t) \to \rho_e$ where $\rho_e$is termed as equilibrium price. Thus, we can say unambiguously that the price-quantity equilibrium is

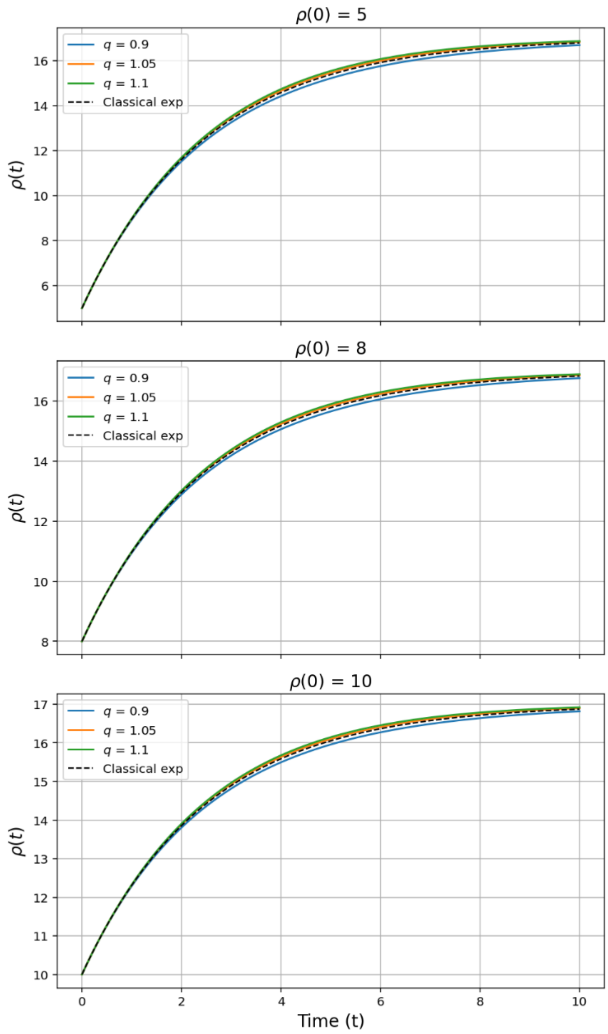

Figure 1 **Plot of $\rho(t)$ vs t for Different Initial Prices and q. The Dotted Line Shows the Classical Plot Whereas Colored Lines Are for Different q-Values as Indicated in Inset**

2.2 Inflationary Model

Inflation is defined as a persistent and widespread increase in the overall price level of goods and services in an economy. The foundational idea about the growth of the money supply is the principal cause of inflation which was prominently described by the 18th-century philosopher David Hume. This principle, known as the 'Quantity Theory of Money', posits a direct relationship between the amount of money in an economy and the level of prices. A classic formulation of this theory is represented by the equation

$$\mu(t) = \lambda\pi(t)O(t)$$

... (29)

In this expression, $\mu(t)$ denotes the money supply, $\pi(t)$ is the general price level, and $O(t)$ represents real output at time t. The parameter λ is a constant reflecting the proportion of nominal income $[\pi(t)O(t)]$ that economic agents wish to hold as money balances. The theory was later refined and popularized in the modern era by Milton Friedman by considering the detailed specification of the demand for money. He expressed real money balances $[\mu(t)/\pi(t)]$ as a function of several key variables (see ref. [14-15]). This framework advances the simple Quantity Theory by explicitly modeling the demand for money as a function of its opportunity costs and broader economic determinants. A dynamic representation of inflation can be formulated through a linear ordinary differential equation with constant coefficients, expressed as follows

$$(D_1 - D_2)\pi(t) = 0. \quad \text{... (30)}$$

Where,

$$D_1 = \left(\alpha_0 \frac{d^m}{dt^m} + \alpha_1 \frac{d^{m-1}}{dt^{m-1}} + \cdots + \alpha_m\right), \quad \text{... (31)}$$

$$D_2 = \beta_0 \frac{d^n}{dt^n} + \beta_1 \frac{d^{n-1}}{dt^{n-1}} + \cdots + \beta_n. \quad \text{... (32)}$$

and $\alpha's$ and $\beta's$ are constants. In order to obtain conclusions economists have long sought to quantify the impact of changes of quantity of money on prices. The conventional mathematical toolkit for this analysis consists of impulse response functions and comparative statics. This paper introduces a third, often overlooked approach. Here, we use q-Laplace transform as a fresh perspective on this classic problem. Now, we analyze the system by changing the domain using the Laplace transform. At first, we use by $\mathcal{L}_q\{\mu(t)\} = \mu_q(s)$ and $\mathcal{L}_q\{\pi(t)\} = \mathcal{G}_q(s)$. Then, for simplicity, we set initial conditions to zero which reveals a core relationship $T_q(s) = \mathcal{G}_q(s)/\mu_q(s)$. This $T_q(s)$ is basically named as transfer function. Using q-Laplace' transform in (29), and (30), $T_q(s)$ can be written as,

$$T_q(s) = \frac{\beta_0 s^n + \beta_1 s^{n-1} + \cdots + \beta_{n-1}s + \beta_n}{\alpha_0 s^m + \alpha_1 s^{m-1} + \cdots + \alpha_{m-1}s + \alpha_m} \quad \text{... (33)}$$

which is basically the ratio of polynomials in s. Thus, $T_q(s)$ is fully encapsulates the dynamic link between money and prices. The power of this technique lies on its generality and for any hypothetical change in the money supply $\mu_q(s)$, the resulting price path $\pi(t)$can be determined. features and dynamics of a particular economy. However, if we suppose the initial problem as,

$$\frac{d_q^2\pi(t)}{d_q t^2} + c\frac{d_q\pi(t)}{d_q t} + f\pi(t) = \mu(t) \quad \text{... (34)}$$

with, $\pi(0) = 0, \pi'(0) = 0$. Applying Laplace' transform on both sides of the equation, we can write,

$$T_q(s) = \frac{\mathcal{G}_q(s)}{\mu_q(s)} = \frac{1}{s^2 + cs + f} \quad \text{... (35)}$$

Hence, we consider three different situations. At first, in new q-differential form (35) can be written as,

$$\mathcal{G}_q(s) = \frac{1}{s(s^2 + cs + f)}. \quad \text{... (36)}$$

Applying partial fraction method and concept of q-Laplace' transform we obtain,

$$\pi_q(t) = \frac{1}{f} - \frac{e_q(\alpha_+ t)}{\alpha_+(\alpha_- - \alpha_+)} + \frac{e_q(\alpha_- t)}{\alpha_-(\alpha_- - \alpha_+)} \quad \text{... (37)}$$

Here,$\alpha_\pm = \frac{-c\pm\sqrt{c^2-4f}}{2}$.Substituting the value of α_+, α_-, we obtain, $\pi_q(t \to \infty) = \frac{1}{f}$. Thus, in this example, it is shown that, the price level of the economy returns to its steady-state value after a rise in the quantity of money. Secondly, if we consider s, the quantity of money in the economy is increased by a constant amount c with time and we model $\mu(t) = ct$, then $\mu_q(s) = c\, q^2/s^2$, then,

$$\mathcal{G}_q(s) = cq^2 \frac{1}{s^2(s^2 + cs + f)}$$
$$= cq^2\left[\frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s-\alpha_+)} + \frac{D}{(s-\alpha_-)}\right],$$

where, $A = \frac{\alpha_+ + \alpha_-}{\alpha_+^2\alpha_-^2}$, $B = \frac{1}{\alpha_+\alpha_-}$, $C = \frac{1}{\alpha(\alpha_+ - \alpha_-)}$, $D = -\frac{1}{\alpha_-^2(\alpha_+ - \alpha_-)}$ respectively. Now, applying inverse Laplace' transform, we obtain,

$$\pi(t) = cq^2\left[A + \frac{Bt}{q^2} + Ce_q(\alpha_+ t) + De_q(\alpha_- t)\right] \quad \text{... (38)}$$

Where, $A, B, C,$ and D can be determined for particular initial value problem. If we consider the initial value problem as

$$\frac{d_q^2\pi(t)}{d_q t^2} + 6\frac{d_q\pi(t)}{d_q t} + 8\pi(t) = \mu(t) \quad \text{... (39)}$$

with, $\pi(0) = 0, \pi'(0) = 0$.

$$T_q(s) = \frac{\mathcal{G}_q(s)}{\mu_q(s)} = \frac{1}{s^2 + 6s + 8}$$

And the solution will be

$$\pi(t) = \frac{1}{8} - \frac{1}{2} e_q(-2t) + \frac{1}{4} e_q(-4t)$$

... (40)

Thus, for very large value of t, the price level $\pi(t)$ will reach the steady value. Hence, even if the quantity of money increases, the level of economy no doubt returns to its steady-state value.

2.3 Model of Firm Sponsored Job Training

It is well established that the continuous enhancement of employee competencies constitutes a critical determinant of organizational productivity and sustainability. In contemporary corporate environments, structured programs aimed at job-specific skill development that may evolved into an indispensable component of human resource management. Systematic training interventions not only improve individual efficiency but also contribute to the firm's aggregate output and adaptability in competitive markets. In ref. [18], the short-run output dynamics of a firm is described elaborately that shows the implementation off-the-job training, wherein employees receive instruction away from their usual workstations. The study presents a formal analytical framework for a single-product firm operating in the short run, where employee training is conducted through an external specialized agency on a batch-wise and continuous basis. The firm incurs the full cost of such training programs, which are assumed to be uniform and non-discriminatory across participants. We suppose, η denote the total workforce, l represent the number of employees currently engaged in training, and $\mathscr{s}$ indicate those not participating in training within a given batch. These parameters $\eta, l,$and $\mathscr{s}$ are treated as implicit functions of time, reflecting the dynamic nature of labour allocation between training and production. The firm's production function is characterized by decreasing returns to scale**,** with labour serving as the sole input factor in the short-run production process. Using the Laplace transform approach, the long-run output behaviour of the firm in this framework can be derived analytically. In the long-run scenario, the firm's production function is assumed to exhibit constant returns to scale, and expressed as $f(\eta) = \gamma\eta_1$where γ represents a proportionality constant that captures the overall productivity level of the workforce. This formulation implies that in the long run, the firm's output increases linearly with the size of its effective labour

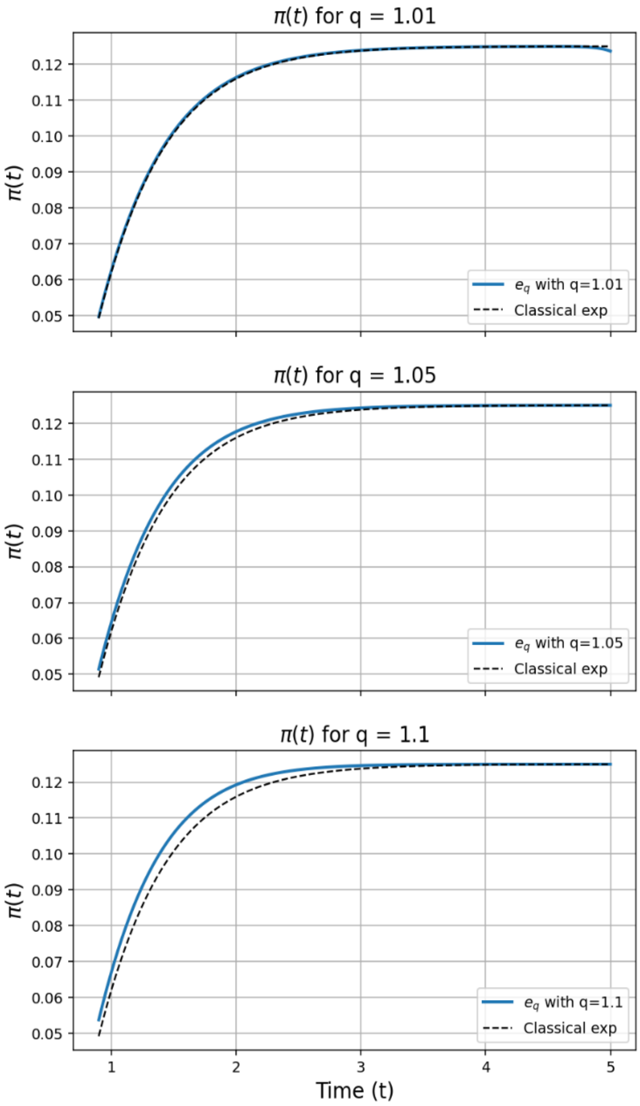

Figure 2 Plot of Price Level with t for Different q-Values

force, reflecting the balancing effect of learning and experience on production efficiency. Thus, in an ongoing training program, the q-output by the function $Y_q(\eta)$ with the restriction $Y_q(0) = 0$ is given by,

$$Y_q(\eta) = \xi sin_q(\lambda\eta)$$

... (41)

3. Comparative Analysis

In this section we compare the three q-model with their classical counterpart by considering arbitrary dataset. Note that, in figure 1, we plot the classical $\rho(t)$ with t where initial price $\rho(0)$ is supposed to be 5\$, 8\$, and 10 \$ whereas, $a_1 = 95$, $a_2 = 2$, $a_3 = 10$, $a_4 = 3$, and $\lambda' = 0.4$. The equilibrium price,

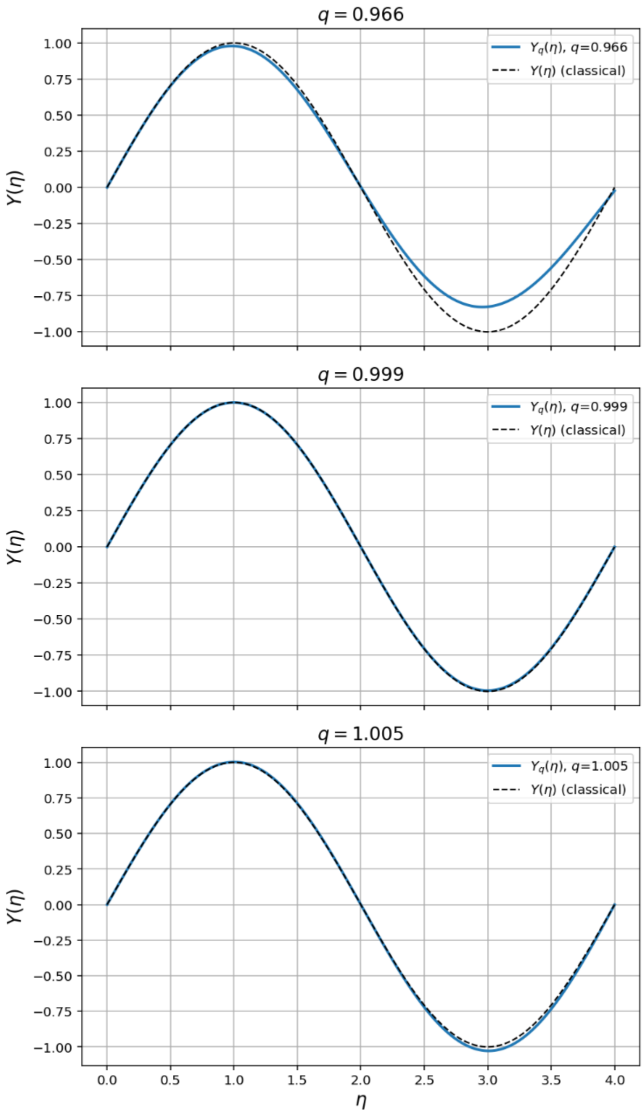

Figure 3 Plot Shows q-output with Total Workforce For Different q-values

$\rho_e = 17$ when $\delta(t) = \sigma(t) = 61$. Figure 1 shows, after a certain time interval, on the basis of supply and demand $\rho(t)$ reaches the equilibrium. Most importantly, here we consider 50 terms in calculating the q-exponential out of infinite terms. In figure 2, we plot price level with $\pi(t)$ with t that indicates the saturation of price levels occurs at different time for different values of q. Deviation of q-value from 1 does not affect the saturation value largely but it shows notable variation for a particular timespan. In addition, we consider the positive values of price level only for convenience. Here also the dotted curves show classical values whereas colored lines indicate the variation of price level for different values of q. In figure 3, output of a training program is plotted with the total workforce. As like as the classical example (as described in ref. 15), the q-model also predict the output in terms of deformed sinusoidal function. The q-plot shows, for large workforce the total output differs largely from the classical values. It hints there may be some other parameters to be included in time of developing the model. In another way, the introduction of q, may predict the output in case-to-case basis by finding the values of q for a particular farm-training scenario.

Conclusion

In this paper, we study the Walrasian model, Inflationary model, and Farm Sponsored Job Training Model in view of q-calculus. This methodology enriches the analytical scope of q-modelling by introducing a bridge between purely discrete and purely continuous mathematical scenario. In this q-framework there is a scope to find out scale-dependent parameter adjustments, where the parameters can change their values either smoothly or stepwise whereas it needs AI driven data analysis machinery. In this way, the q-calculus approach extends the limits of classical models into a more versatile framework, better suited for analyzing the complexities of modern economic systems. Most importantly, when $q \to 1$, (28), (38). (39), (41) reduces to the exact classical from (as described in ref. [15]) that depicts the validity of this q-model with its corresponding classical model. Moreover, this q-framework can also add a good pedagogical example in mathematics as well as mathematical economics.

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