



Special Issue of Second International Conference on Science and Technology (ICOST 2021)

Pentapartitioned Neutrosophic Pythagorean Set

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Abstract

The aim of this paper is to introduce the concept of Pentapartitioned Neutrosophic Pythagorean Set with truth membership function T , contradiction membership function C , ignorance membership function U and false membership function F as dependent neutrosophic components and unknown membership function I as an independent neutrosophic component. Pentapartitioned neutrosophic pythagorean set is an extension of Quadripartitioned neutrosophic pythagorean set. By combining five value neutrosophic logic with neutrosophic pythagorean set, we will obtain Pentapartitioned neutrosophic pythagorean set. The complement, union and intersection of Pentapartitioned neutrosophic pythagorean sets are also discussed in this paper. We establish some of its relative properties of Pentapartitioned neutrosophic pythagorean set.

Keywords: Neutrosophic Set, Quadripartitioned Neutrosophic set, Pentapartitioned Neutrosophic Set.

1 Introduction

The fuzzy set was introduced by Zadeh [19] in 1965. The concept of Neutrosophic set was introduced by F. Smarandache which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data.

Smarandache [15] in proposed neutrosophic sets. In neutrosophic sets, the indeterminacy membership function walks along independently of the truth membership or of the falsity membership. Neutrosophic theory has been widely explored by researchers for application purpose in handling real life situations involving uncertainty. Although the hesitation margin of neutrosophic theory is independent of the truth or falsity membership, looks more general than intuitionistic fuzzy sets yet. Recently, in Atanassov et al. [3] studied the relations between inconsistent intuitionistic fuzzy sets, picture fuzzy sets, neutrosophic sets and intuitionistic fuzzy sets; however, it remains in doubt that whether the indeterminacy associated to a particular element occurs due to the

belongingness of the element or the non-belongingness. This has been pointed out by Chatterjee et al. [4] while introducing a more general structure of neutrosophic set viz. quadripartitioned single valued neutrosophic set (QSVNS). The idea of QSVNS is actually stretched from Smarandache's four numerical-valued neutrosophic logic and Belnap's four valued logic, where the indeterminacy is divided into two parts, namely, "unknown" i.e., neither true nor false and "contradiction" i.e., both true and false. In the context of neutrosophic study however, the QSVNS looks quite logical. Also, in their study, Chatterjee [4] et al. analyzed a real-life example for a better understanding of a QSVNS environment and showed that such situations occur very naturally. [1-6]

In 2018 Smarandache [17] generalized the Soft Set to the Hyper Soft Set by transforming the classical uni-argument function F into a multi-argument function:

In 2016, F. Smarandache [14] introduced for the first time the degree of dependence between the components of fuzzy set and neutrosophic sets. The main idea of Neutrosophic sets is to characterize each value statement in a 3D – Neutrosophic space, where each dimension of the space represents respectively the truth membership, falsity membership and the indeterminacy, when two components T and F are dependent and I is independent then $T+I+F \leq 2$.

Rama Malik and Surpati Pramanik [12] introduced Pentapartitioned neutrosophic set and its properties. Here indeterminacy is divided into three parts as contradiction, ignorance and unknown membership function. [7-10]

If T and F are dependent neutrosophic pythagorean components then $T^2 + F^2 \leq 1$. Similarly, for U and C as dependent neutrosophic pythagorean components then $C^2 + U^2 \leq 1$. When combining both we get Quadripartitioned pythagorean set with dependent components as $T^2 + F^2 + C^2 + U^2 \leq 2$

R. Radha and A. Stanis Arul Mary [10] introduced Quadripartitioned neutrosophic pythagorean sets with T, C, U, F as dependent neutrosophic components.

In this we have to introduce the concept of introduced the concept of Pentapartitioned neutrosophic pythagorean set with T, C, U and F are dependent neutrosophic components and I as an independent neutrosophic components and establish some of its properties. [11-18]

2. Preliminaries

2.1 Definition [15]

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$$

Where $T_A, I_A, F_A: U \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy and $F_A(x)$ is the degree of non-membership.

Here, $T_A(x)$ and $F_A(x)$ are dependent neutrosophic components and $I_A(x)$ is an independent component.

2.2 Definition [4]

Let X be a universe. A Quadripartitioned neutrosophic set A with independent neutrosophic

components on X is an object of the form

$$A = \{ \langle x, T_A(x), C_A(x), U_A(x), F_A(x) \rangle : x \in X \}$$

$$\text{and } 0 \leq T_A(x) + C_A(x) + U_A(x) + F_A(x) \leq 4$$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership and $F_A(x)$ is the false membership.

2.3 Definition [12]

Let P be a non-empty set. A Pentapartitioned neutrosophic set A over P characterizes each element p in P a truth -membership

function T_A , a contradiction membership function C_A , an ignorance membership function G_A , unknown membership function U_A and a false membership function F_A , such that for each p in P

$$T_A + C_A + G_A + U_A + F_A \leq 5$$

2.4 Definition [10]

Let X be a universe. A Quadripartitioned neutrosophic pythagorean set A with dependent neutrosophic components A on X is an object of the form

$$A = \{ \langle x, T_A(x), C_A(x), U_A(x), F_A(x) \rangle : x \in X \}$$

Where $T_A + F_A \leq 1, C_A + U_A \leq 1$ and $0 \leq$

$$(T_A(x))^2 + (C_A(x))^2 + (U_A(x))^2 + (F_A(x))^2 \leq 2$$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership and $F_A(x)$ is the false membership.

2.5 Definition [10]

A Quadripartitioned neutrosophic pythagorean set A is contained in another Quadripartitioned neutrosophic pythagorean set B (i.e) $A \subseteq B$ if

$$T_A(x) \leq T_B(x), C_A(x) \leq C_B(x), U_A(x) \leq U_B(x)$$

$$\text{and } F_A(x) \leq F_B(x)$$

2.6 Definition [10]

The complement of a Quadripartitioned neutrosophic pythagorean set (K, A) on X Denoted by $(K, A)^c$ and is defined as

$$K^c(x) = \{ \langle x, F_A(x), U_A(x), C_A(x), T_A(x) \rangle : x \in X \}$$

2.7 Definition [10]

Let X be a non-empty set,

$$A = \langle x, T_A(x), C_A(x), U_A(x), F_A(x) \rangle \text{ and}$$

$$B = \langle x, T_B(x), I_B(x), F_B(x) \rangle \text{ are}$$

Quadripartitioned neutrosophic pythagorean sets. Then

$$A \cup B = \langle x, \max(T_A(x), T_B(x)),$$

$$\max(C_A(x), C_B(x)),$$

$$\min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)) \rangle$$

$$A \cap B = \langle x, \min(T_A(x), T_B(x)),$$

$$\min(C_A(x), C_B(x)), \max(U_A(x), U_B(x)), \max(F_A(x), F_B(x)) >$$

2.8 Definition [10]

A Quadripartitioned neutrosophic pythagorean set (K, A) over the universe X is said to be empty neutrosophic pythagorean set with respect to the parameter A if

$$T_{K(e)} = 0, C_{K(e)} = 0, U_{K(e)} = 1, F_{K(e)}=1, \forall x \in X, \forall e \in A. \text{ It is denoted by } 0_N$$

2.9 Definition [10]

A Quadripartitioned neutrosophic pythagorean set (K, A) over the universe X is said to be universe neutrosophic pythagorean set with respect to the parameter A if

$$T_{K(e)} = 1, C_{K(e)} = 1, U_{K(e)} = 0, F_{K(e)}=0, \forall x \in X, \forall e \in A. \text{ It is denoted by } 1_N$$

3. Pentapartitioned Neutrosophic Pythagorean Set (PNPS or PNP Set)

3.1 Definition

Let X be a universe. A Pentapartitioned neutrosophic pythagorean set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

$$A = \{ \langle x, T_A, C_A, I_A, U_A, F_A \rangle : x \in X \}$$

Where $T_A + F_A \leq 1, C_A + U_A \leq 1$ and $(T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A)^2 \leq 3$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership and $I_A(x)$ is an unknown membership.

3.2 Definition

A Pentapartitioned neutrosophic pythagorean set A is contained in another Pentapartitioned neutrosophic pythagorean soft set B (i.e) $A \subseteq B$ if $T_A \leq T_B, C_A \leq C_B, I_A \geq I_B, U_A \geq U_B$ and $F_A \geq F_B$

3.3 Definition

The complement of a Pentapartitioned neutrosophic pythagorean set (F, A) on X denoted by $(K, A)^c$ and is defined as

$$K^c(x) = \{ \langle x, F_A, U_A, 1 - I_A, C_A, T_A \rangle : x \in X \}$$

3.4 Definition

Let X be a non-empty set, $A = \langle x, T_A, C_A, I_A, U_A, F_A \rangle$ and $B = \langle x, T_B, C_B, I_B, U_B, F_B \rangle$ are two Pentapartitioned neutrosophic pythagorean sets.

Then

$$A \cup B = \langle x, \max(T_A, T_B), \max(C_A, C_B), \min(I_A, I_B), \min(U_A, U_B), \min(F_A, F_B) \rangle$$

$$A \cap B = \langle x, \min(T_A, T_B), \min(C_A, C_B), \max(I_A, I_B), \max(U_A, U_B), \max(F_A, F_B) \rangle$$

3.5 Definition

A Pentapartitioned neutrosophic pythagorean set (K, A) over the universe X is said to be empty Pentapartitioned neutrosophic pythagorean set \emptyset with respect to the parameter A if

$$T_{K(e)} = 0, C_{K(e)} = 0, I_{K(e)} = 1, U_{K(e)} = 1, F_{K(e)}=1, \forall x \in X, \forall e \in A. \text{ It is denoted by } \emptyset$$

3.6 Definition

A Pentapartitioned neutrosophic pythagorean set (K, A) over the universe X is said to be Δ universe Pentapartitioned neutrosophic pythagorean set with respect to the parameter A if

$$T_{K(e)} = 1, C_{K(e)} = 1, I_{K(e)} = 0, U_{K(e)} = 0, F_{K(e)}=0, \forall x \in X, \forall e \in A. \text{ It is denoted by } \Delta$$

3.7 Definition

Let A and B be two Pentapartitioned neutrosophic pythagorean sets on X then $A \setminus B$ may be defined as $A \setminus B = \langle x, \min(T_A, F_B), \min(C_A, U_B), \max(I_A, 1 - I_B), \max(U_A, C_B), \max(F_A, T_B) \rangle$

3.8 Definition

K_E is called absolute Pentapartitioned neutrosophic pythagorean set over X if $K(e) = \Delta$ for any $e \in E$. We denote it by X_E

3.9 Definition

K_E is called relative null Pentapartitioned neutrosophic pythagorean set over X if $K(e) = \emptyset$ for any $e \in E$. We denote it by \emptyset_E . Obviously $\emptyset_E = X_E^c$ and $X_E = \emptyset_E^c$

3.10 Definition

The complement of a Pentapartitioned neutrosophic pythagorean set (K, A) over X can also be defined as $(K, A)^c = X_E \setminus K(e)$ for all $e \in A$.

Note: We denote X_E by X in the proofs of proposition.[19-23]

3.11 Definition

If (K, A) and (L, B) be two Pentapartitioned neutrosophic pythagorean set then "(K, A) AND (L, B)" is denoted by $(K, A) \wedge (L, B)$ and is defined by $(K, A) \wedge (L, B) = (H, A \times B)$ where $H(a, b) = K(a) \cap L(b) \forall a \in A$ and $\forall b \in B$, where \cap is the operation intersection of Pentapartitioned neutrosophic pythagorean set.

3.12 Definition

If (K, A) and (L, B) be two Pentapartitioned neutrosophic pythagorean set then “ (K, A) OR (L, B) ” is denoted by $(K, A) \vee (L, B)$ and is defined by $(K, A) \vee (L, B) = (K, A \times B)$ where $K(a, b) = K(a) \cup L(b) \forall a \in A$ and $\forall b \in B$, where \cup is the operation union of Pentapartitioned neutrosophic pythagorean set.

3.13 Theorem

Let (K, A) and (L, A) be Pentapartitioned neutrosophic pythagorean set over the universe X . Then the following are true.

- (i) $(K, A) \subseteq (L, A)$ iff $(K, A) \cap (L, A) = (K, A)$
- (ii) $(K, A) \subseteq (L, A)$ iff $(K, A) \cup (L, A) = (L, A)$

Proof:

(i) Suppose that $(K, A) \subseteq (L, A)$, then $K(e) \subseteq L(e)$ for all $e \in A$.
 Let $(K, A) \cap (L, A) = (H, A)$.
 Since $H(e) = K(e) \cap L(e) = K(e)$ for all $e \in A$, by definition $(H, A) = (K, A)$.
 Suppose that $(K, A) \cap (L, A) = (K, A)$.
 Let $(K, A) \cap (L, A) = (H, A)$.
 Since $H(e) = K(e) \cap L(e) = K(e)$ for all $e \in A$, we know that $K(e) \subseteq L(e)$ for all $e \in A$.
 Hence $(K, A) \subseteq (L, A)$.

(ii) The proof is similar to (i).

3.14 Theorem

Let (F, A) , (G, A) , (H, A) , and (S, A) are Pentapartitioned neutrosophic pythagorean set over the universe X . Then the following are true.

- (i) If $(F, A) \cap (G, A) = \emptyset_A$, then $(F, A) \subseteq (G, A)^c$
- (ii) If $(F, A) \subseteq (G, A)$ and $(G, A) \subseteq (H, A)$ then $(F, A) \subseteq (H, A)$
- (iii) If $(F, A) \subseteq (G, A)$ and $(H, A) \subseteq (S, A)$ then $(F, A) \cap (H, A) \subseteq (G, A) \cap (S, A)$
- (iv) $(F, A) \subseteq (G, A)$ iff $(G, A)^c \subseteq (F, A)^c$

Proof:

(i) Suppose that $(F, A) \cap (G, A) = \emptyset_A$.
 Then $F(e) \cap G(e) = \emptyset$.
 So, $F(e) \subseteq \cup G(e) = G^c(e)$ for all $e \in A$. Therefore, we have $(F, A) \subseteq (G, A)^c$.

Proof of (ii) and (iii) are obvious.

- (iv) suppose that $(F, A) \subseteq (G, A)$
 $\Leftrightarrow F(e) \subseteq G(e)$ for all $e \in A$.
 $\Leftrightarrow (G(e))^c \subseteq (F(e))^c$ for all $e \in A$.
 $\Leftrightarrow G^c(e) \subseteq F^c(e)$ for all $e \in A$.
 $\Leftrightarrow (G, A)^c \subseteq (F, A)^c$

3.15 Definition

Let I be an arbitrary index $\{(F_i, A)\}_{i \in I}$ be a subfamily of Pentapartitioned neutrosophic pythagorean set over the universe X .

- (i) The union of these Pentapartitioned neutrosophic pythagorean set is the Pentapartitioned neutrosophic pythagorean set (H, A) where $H(e) = \cup_{i \in I} F_i(e)$ for each $e \in A$.

We write $\cup_{i \in I} (F_i, A) = (H, A)$

- (ii) The intersection of these Pentapartitioned neutrosophic pythagorean set is the Pentapartitioned neutrosophic pythagorean set (M, A) where $M(e) = \cap_{i \in I} F_i(e)$ for each $e \in A$.

We write $\cap_{i \in I} (F_i, A) = (M, A)$

3.16 Theorem

Let I be an arbitrary index set and $\{(F_i, A)\}_{i \in I}$ be a subfamily of Pentapartitioned neutrosophic pythagorean set over the universe X . Then

- (i) $(\cup_{i \in I} (F_i, A))^c = \cap_{i \in I} (F_i, A)^c$
- (ii) $(\cap_{i \in I} (F_i, A))^c = \cup_{i \in I} (F_i, A)^c$

Proof:

(i) $(\cup_{i \in I} (F_i, A))^c = (H, A)^c$,
 By definition $H^c(e) = X_E \setminus H(e) = X_E \setminus \cup_{i \in I} F_i(e) = \cap_{i \in I} (X_E \setminus F_i(e))$ for all $e \in A$.

On the other hand, $(\cap_{i \in I} (F_i, A))^c = (K, A)$.

By definition, $K(e) = \cap_{i \in I} F_i^c(e) = \cap_{i \in I} (X - F_i(e))$ for all $e \in A$.

(ii) It is obvious.

Note: We denote \emptyset_E by \emptyset and X_E by X .

3.17 Theorem

Let (K, A) be Pentapartitioned neutrosophic pythagorean set over the universe X . Then the following are true.

- (i) $(\emptyset, A)^c = (X, A)$
- (ii) $(X, A)^c = (\emptyset, A)$

Proof:

(i) Let $(\emptyset, A) = (K, A)$

Then $\forall e \in A$,

$$K(e) = \{ \langle x, T_{K(e)}, C_{K(e)}, I_{K(e)}, U_{K(e)}, F_{K(e)} \rangle : x \in X \}$$

$$= \{ \langle x, 0, 0, 1, 1, 1 \rangle : x \in X \}$$

Thus $(\emptyset, A)^c = (K, A)^c$

Then $\forall e \in A$,

$$(K(e))^c = \{ \langle x, T_{K(e)}, C_{K(e)}, I_{K(e)}, U_{K(e)}, F_{K(e)} \rangle : x \in X \}^c$$

$$= \{ \langle x, F_{K(e)}, U_{K(e)}, 1 - I_{K(e)}, C_{K(e)}, T_{K(e)} \rangle : x \in X \}$$

$$= \{ \langle x, 1, 1, 0, 0, 0 \rangle : x \in X \} = X$$

Thus $(\emptyset, A)^c = (X, A)$

(ii) Proof is similar to (i)

3.18 Theorem

Let (K, A) be Pentapartitioned neutrosophic pythagorean set over the universe X . Then the following are true.

- (i) $(K, A) \cup (\emptyset, A) = (K, A)$
- (ii) $(K, A) \cup (X, A) = (X, A)$

Proof:

(i) $(K, A) = \{ e, \langle x, T_{K(e)}, C_{K(e)}(x), I_{K(e)}, U_{F(e)}, F_{F(e)} \rangle : x \in X \} \forall e \in A$

$$\begin{aligned}
 (\emptyset, A) &= \{e, (x, 0, 0, 1, 1, 1) : x \in X\} \forall e \in A \\
 (K, A) \cup (\emptyset, A) &= \{e, (x, \max(T_{K(e)}, 0), \\
 &\max(C_{K(e)}, 0), \min(I_{K(e)}, 1), \min(U_{K(e)}, 1) \\
 &\min(F_{K(e)}, 1))\} \forall e \in A \\
 &= \{e, (x, T_{K(e)}, C_{K(e)}, I_{K(e)}, U_{K(e)}, F_{K(e)})\} \\
 &\quad \forall e \in A \\
 &= (K, A)
 \end{aligned}$$

(ii) Proof is similar to (i).

3.19 Theorem

Let (K, A) be Pentapartitioned neutrosophic pythagorean set over the universe X. Then the following are true.

- (i) $(K, A) \cap (\emptyset, A) = (\emptyset, A)$
- (ii) $(K, A) \cap (X, A) = (K, A)$

Proof:

$$\text{Let } (K, A) = \{e, (x, T_{K(e)}, C_{K(e)}, U_{K(e)}, F_{K(e)})\}$$

$$\begin{aligned}
 \text{and } (\emptyset, A) &= \{e, (x, 0, 0, 1, 1, 1) : x \in X\} \forall e \in A \\
 (K, A) \cap (\emptyset, A) &= \{e, (x, \min(T_{K(e)}, 0), \min(C_{K(e)}, 0), \\
 &\max(I_{K(e)}, 1), \max(U_{K(e)}, 1), \max(F_{K(e)}, 1))\} \\
 &\quad \forall e \in A \\
 &= \{e, (x, 0, 0, 1, 1, 1)\} \forall e \in A \\
 &= (\emptyset, A) \\
 \text{So, } (K, A) \cap (\emptyset, A) &= (\emptyset, A).
 \end{aligned}$$

3.20 Theorem

Let (K, A) and (L, B) are Pentapartitioned neutrosophic pythagorean set over the universe X. Then the following are true.

- (i) $(K, A) \cup (\emptyset, B) = (K, A)$ iff $B \subseteq A$
- (ii) $(K, A) \cup (X, B) = (X, A)$ iff $A \subseteq B$

Proof:

We have for (K, A),
 $K(e) = \{(x, T_K, C_K, I_K, U_K, F_K) : x \in X\} \forall e \in A$

Also let $(\emptyset, B) = (L, B)$, then
 $L(e) = \{(x, 0, 0, 1, 1, 1) : x \in X\} \forall e \in B$
 Let $(K, A) \cup (\emptyset, B) = (K, A) \cup (L, B) = (H, C)$
 where $C = A \cup B$ and for all $e \in C$

H(e) may be defined as
 $H(e) = K(e)$ if $e \in A - B$,
 $= L(e)$ if $e \in B - A$
 $= K(e) \cup L(e)$ if $e \in A \cap B$

Here, $K(e) \cup L(e) = K(e)$. Then
 $H(e) = K(e)$ if $e \in A - B$,
 $= L(e)$ if $e \in B - A$
 $= K(e)$ if $e \in A \cap B$

Let $B \subseteq A$

Then $H(e) = K(e)$ if $e \in A - B$
 $= K(e)$ if $e \in A \cap B$

Hence $H(e) = K(e) \forall e \in A$
 Conversely Let $(K, A) \cup (\emptyset, B) = (K, A)$
 Then $A = A \cup B \implies B \subseteq A$
 (ii) Proof is similar to (i)

3.21 Theorem

Let (K, A) and (L, B) are two Pentapartitioned neutrosophic pythagorean set over the universe X. Then the following are true.

- (i) $(K, A) \cap (\emptyset, B) = (\emptyset, A \cap B)$
- (ii) $(K, A) \cap (X, B) = (F, A \cap B)$

Proof:

(i) We have for (K, A)
 $K(e) = \{(x, T_{K(e)}, C_{K(e)}, I_{K(e)}, U_{K(e)}, F_{K(e)}) : x \in X\} \forall e \in A$

Also let $(\emptyset, B) = (L, B)$ then
 $L(e) = \{(x, 0, 0, 1, 1, 1) : x \in X\} \forall e \in B$
 Let $(K, A) \cap (\emptyset, B) = (K, A) \cap (L, B) = (H, C)$
 where $C = A \cap B$ and $\forall e \in C$
 $H(e) = \{(x, \min(T_{K(e)}, T_{L(e)}), \min(C_{K(e)}, C_{L(e)}), \\ \max(I_{K(e)}, I_{L(e)}), \max(U_{K(e)}, U_{L(e)}), \max(F_{K(e)}, F_{L(e)}))\} \\ : x \in X$
 $= \{(x, \min(T_{K(e)}, 0), \min(C_{K(e)}, 0), \max(I_{K(e)}, 1), \\ \max(U_{K(e)}, 1), \max(F_{K(e)}, 1)) : x \in X\}$
 $= \{(x, 0, 0, 1, 1, 1) : x \in X\}$
 $= (L, B) = (\emptyset, B)$

Thus $(K, A) \cap (\emptyset, B) = (\emptyset, B) = (\emptyset, A \cap B)$
 (ii) Proof is similar to (i).

3.22 Theorem

Let (K, A) and (L, B) be Pentapartitioned neutrosophic pythagorean set over the universe X. Then the following are true.

- (i) $((K, A) \cup (L, B))^C \subseteq (K, A)^C \cup (L, B)^C$
- (ii) $(K, A)^C \cap (L, B)^C \subseteq ((K, A) \cap (L, B))^C$

3.23 Theorem

Let (K, A) and (G, A) are two Pentapartitioned neutrosophic pythagorean sets over the same universe X. We have the following

- (i) $((K, A) \cup (G, A))^C = (K, A)^C \cap (G, A)^C$
- (ii) $((K, A) \cap (G, A))^C = (K, A)^C \cup (G, A)^C$

Proof:

(i) Let $(K, A) \cup (G, A) = (H, A) \forall e \in A$
 $H(e) = K(e) \cup G(e)$
 $= \{(x, \max(T_{K(e)}, T_{G(e)}), \max(C_{K(e)}, C_{G(e)}), \\ \min(I_{K(e)}, I_{G(e)}), \min(U_{K(e)}, U_{G(e)}), \\ \min(F_{K(e)}, F_{G(e)}))\}$

Thus $(K, A) \cup (G, A))^C = (H, A)^C \forall e \in A$
 $(H(e))^C = (K(e) \cup G(e))^C$
 $= \{(x, \max(T_{K(e)}, T_{G(e)}), \max(C_{K(e)}, C_{G(e)}), \\ \min(I_{K(e)}, I_{G(e)}), \min(U_{K(e)}, U_{G(e)}),$

$$\min(F_{K(e)}, F_{G(e)})\}^C$$

$$= \{(x, \min(F_{K(e)}, F_{G(e)}), \min(U_{K(e)}, U_{G(e)}),$$

$$\max(1 - I_{K(e)}, 1 - I_{G(e)}), \max(C_{K(e)}, C_{G(e)}),$$

$$\max(T_{K(e)}, T_{G(e)})\}$$
 Again $(K, A)^C \cap (G, A)^C = (I, A)$ where $\forall e \in A$
 $I(e) = (K(e))^C \cap (G(e))^C$

$$= \{(x, \min(F_{K(e)}, F_{G(e)}), \min(U_{K(e)}, U_{G(e)}),$$

$$\max(1 - I_{K(e)}, 1 - I_{G(e)}), \max(C_{K(e)}, C_{G(e)}),$$

$$\max(T_{K(e)}, T_{G(e)})\}$$
 Thus $((K, A) \cup (G, A))^C = (K, A)^C \cap (G, A)^C$
 (ii) Let $(K, A) \cap (G, A) = (H, A) \forall e \in A$
 $H(e) = F(e) \cap G(e)$

$$= \{(x, \min(T_{K(e)}, T_{G(e)}), \min(C_{K(e)}, C_{G(e)}),$$

$$\max(I_{K(e)}, I_{G(e)}), \max(U_{K(e)}, U_{G(e)}),$$

$$\max(F_{K(e)}, F_{G(e)})\} \forall e \in A$$
 Thus $(K, A) \cap (G, A)^C = (H, A)^C$
 $(H(e))^C = (K(e) \cap G(e))^C$

$$= \{(x, \min(T_{K(e)}, T_{G(e)}), \min(C_{K(e)}, C_{G(e)}),$$

$$\max(I_{K(e)}, I_{G(e)}), \max(U_{K(e)}, U_{G(e)}),$$

$$\max(F_{K(e)}, F_{G(e)})\}^C$$

$$= \{(x, \min(T_{K(e)}, T_{G(e)}), \min(U_{K(e)}, U_{G(e)}),$$

$$\max(I_{K(e)}(x), I_{G(e)}(x)), \max(C_{K(e)}, C_{G(e)})$$

$$\max(F_{K(e)}(x), F_{G(e)}(x))\}^C$$

$$= \{(x, \max(F_{K(e)}, F_{G(e)}), \max(U_{K(e)}, U_{G(e)}),$$

$$\min(1 - I_{F(e)}, 1 - I_{G(e)}), \min(C_{K(e)}, C_{G(e)}),$$

$$\min(T_{K(e)}, T_{G(e)})\} \forall e \in A$$
 Again $(K, A)^C \cup (G, A)^C = (I, A)$ where $\forall e \in A$
 $I(e) = (K(e))^C \cup (G(e))^C$

$$= \{(x, \max(F_{K(e)}, F_{G(e)}), \max(U_{K(e)}, U_{G(e)}),$$

$$\min(1 - I_{K(e)}, 1 - I_{G(e)}), \min(C_{K(e)}, C_{G(e)}),$$

$$\min(T_{F(e)}, T_{G(e)})\} \forall e \in A$$
 Thus $((K, A) \cap (G, A))^C = (K, A)^C \cup (G, A)^C$

3.24 Theorem

Let (K, A) and (L, A) are two Pentapartitioned neutrosophic pythagorean sets over the same universe X . We have the following

- (i) $((K, A) \wedge (L, A))^C = (K, A)^C \vee (L, A)^C$
- (ii) $((K, A) \vee (L, A))^C = (K, A)^C \wedge (L, A)^C$

Proof:

Let $(K, A) \wedge (L, B) = (H, A \times B)$ where
 $H(a, b) = K(a) \cap L(b) \forall a \in A$ and $\forall b \in B$ where
 \cap is the operation intersection of PNPS.
 Thus $H(a, b) = K(a) \cap L(b)$

$$= \{(x, \min(T_{K(a)}, T_{L(b)}), \min(C_{K(a)}, C_{L(b)}),$$

$$\max(I_{K(a)}, I_{L(b)}), \max(U_{K(a)}, U_{L(b)}),$$

$$\max(F_{K(a)}, F_{L(b)})\}$$

$$((K, A) \wedge (L, B))^C = (H, A \times B)^C$$

$$\forall (a, b) \in A \times B$$
 Thus $(H(a, b))^C = \{(x, \min(T_{K(a)}, T_{L(b)}),$

$$\min(C_{K(a)}, C_{L(b)}), \max(I_{K(a)}, I_{L(b)}),$$

$$\max(U_{K(a)}, U_{L(b)}), \max(F_{K(a)}, F_{L(b)})\}^C$$

$$= \{(x, \max(F_{K(a)}, F_{L(b)}), \max(U_{K(a)}, U_{L(b)}),$$

$$\min(1 - I_{K(a)}, 1 - I_{L(b)}), \min(C_{K(a)}, C_{L(b)}),$$

$$\min(T_{K(a)}, T_{L(b)})\}$$
 Let $(K, A)^C \vee (L, A)^C = (R, A \times B)$ where
 $R(a, b) = (K(a))^C \cup (L(b))^C \forall a \in A$ and
 $\forall b \in B$ where \cup is the operation union of PNPS.
 $R(a, b) = \{(x, \max(F_{K(a)}, F_{L(b)}),$

$$\max(U_{K(a)}, U_{L(b)}), \min(1 - I_{K(a)}, 1 - I_{L(b)}),$$

$$\min(C_{K(a)}, C_{L(b)}), \min(T_{K(a)}, T_{L(b)})\}$$
 Hence $((K, A) \wedge (L, A))^C = (K, A)^C \vee (L, A)^C$
 Similarly, we can prove (ii)

Conclusions

In this paper, we have introduced Pentapartitioned neutrosophic pythagorean set with T, C, U and F as dependent neutrosophic components and I as an independent neutrosophic component. The Pentapartitioned neutrosophic pythagorean set is an extension of Quadripartitioned neutrosophic pythagorean set with dependent neutrosophic components and Pentapartitioned neutrosophic set. We discussed some of its properties of Pentapartitioned neutrosophic pythagorean set. Future works may compromise of the study of different types of operators on Pentapartitioned neutrosophic pythagorean set dealing with actual problems and implementing them in decision making problems.

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