



## Polar Coding in 5G systems

Prahlad D<sup>1</sup>, Aishwarya Puthuraya<sup>2</sup>, Gayathri Srinivasan<sup>2</sup>, Harsh Kumar<sup>2</sup>, Keerthi N Reddy<sup>2</sup>

<sup>1</sup>Professor, Department of Electronics and Communication Engineering, PES University, Karnataka, Bangalore, India.

<sup>2</sup>Department of Electronics and Communication Engineering, PES University, Karnataka, Bangalore, India.

Email: [prahlad@pes.edu](mailto:prahlad@pes.edu)

### Article History

Received: 26 April 2023

Accepted: 21 May 2023

### Keywords:

Polar codes;  
successive cancellation  
decoder;  
successive cancellation list  
decoding;  
repetition;  
puncturing;  
shortening

### Abstract

Polar Coding is a type of line coding that offers good error correction performance across a range of coding rates and block lengths. In the case of enhanced mobile broadband communication service (eMBB) for uplink and downlink control information, polar codes are used as channel coding scheme. Two other frameworks foreseen by 5G include massive machine-type communications (mMTC) and ultra-reliable low-latency communications (URLLC). Successive cancellation decoder is most commonly used in the decoding of 5G polar codes. After generating the bit sequence, the process of polar encoding starts, and Successive Cancellation Decoding or Successive Cancellation List Decoding algorithm can be used for decoding the bits. Repetition, puncturing and shortening are the different rate matching techniques that can be used. This paper presents the encoding and decoding chain in 5G systems using MATLAB.

## 1. Introduction

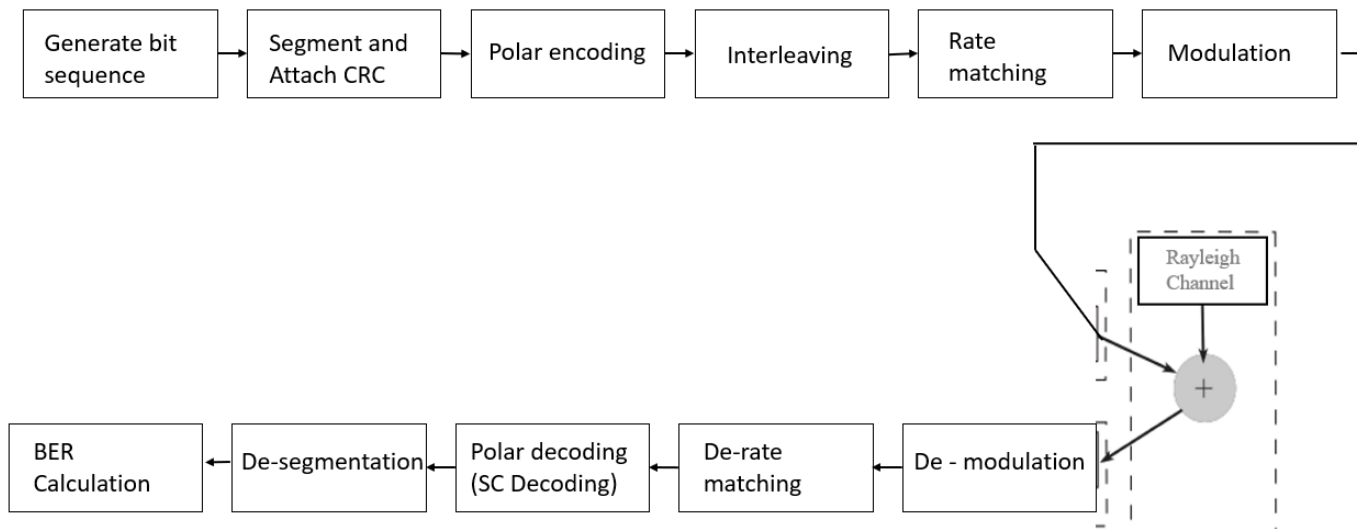
Communication involves transmission of information from the sender to the receiver. There have been multiple iterations of mobile networks introduced in the form of generations, starting from 1G to the proposed 6G technology. (Kamenev, Kameneva, and Kurmaev) Each generation has proved to be faster, efficient, and more resilient than the previous one, but 5G was a pioneer in terms of quality-of-service, where the user is guaranteed service unlike previous generations. A communication system has three parts: a sender, a receiver and a medium. (Arıkan)

The medium is the carrier over which the messages are sent which is often susceptible to noise. Excessive noise levels can result in information loss and data corruption. (Tal and Vardy) Hence, it becomes necessary to control the levels of noise in any communication system. Channel coding is a method which is employed in order to keep the

extent of errors under control. Algorithms used in channel coding are called error correction codes that help in recovering messages sent. (Chen et al.) Polar codes have been used in modern 5G networks since it achieves channel capacity for long block lengths. Figure 1 shows the block diagram of encoding and decoding chain according to 5G standards.

## 2. Literature Review

(Bioglio, Condo, and Land) is a step-by-step guide for understanding and implementing 5G polar code encoding which includes processes such as rate matching, interleaving, etc. The description given includes the individual components of the encoding chain. Basic polarization kernel forms the mathematical foundation of polar codes. Polar codes consider code lengths that are powers of 2, and a code dimension K which is an arbitrary value. The aim of polar codes is to attain satisfactory rate flexibility,



**FIGURE 1. Block Diagram**

low decoding latency and identifying the  $K$  channels that provides the best reliability for transmission of information bits. The Information set includes the indices of the  $K$  channels while the frozen set includes the indices of the remaining  $N-K$  channels. This paper also provides an insight to the Successive Cancellation algorithm which is used for decoding the polar codes in 5G systems.

(Maltiyar and Malviya) depicts Polar codes as a widely accepted choice for the upcoming wireless communication systems. Polar codes with the ability of “low encoding and decoding complexity” achieves the capacity of symmetric channels. Some of the common methods for the decoding of polar codes include belief propagation decoding (BPD) and Successive cancellation decoding (SCD). CRC, a linear block code, achieves better performance when compared to other codes that detect error. (Trifonov and Miloslavskaya)

(Cheng *et al.*) describes CRC-aided PC polar coding scheme which can correct and detect error in a more effective way when compared to Parity-check (PC) polar codes and CRC-assisted polar codes. This is achieved by detecting the error before decoding is completed. (Condo, Ercan, and Gross) The following are the contributions of the paper:

1) Proposal of PC coding scheme that reduces the

rate loss significantly.

2) Required number of PC bits for this scheme.

3) Design of PC bits and CRC location.

Cyclic Redundancy Check bits are present in the middle of the ‘information sequence’ and their previous bits are protected. Therefore, prior to the completion of decoding errors are detected. There should be a limitation on the count of check bits as it impacts rate loss when there are many check bits. (Oliveira and De Lamare)

(Bae) gives an insight of 5G channel coding schemes. For polar codes, the focus remains to be designing an information sequence with a ‘good complexity-to-performance trade-off’. Rate matching techniques that include puncturing, repetition and shortening is an important aspect of design. Delay during decoding is reduced by using distributed CRC. Advanced decoders like list decoders i.e., ‘CRC assisted successive cancellation list decoder’ are used to improve the performance for fixed length code.

### 3. Methodology

#### 3.1. Generate the bit sequence

Generate the bit sequence of the desired payload size with a codeword length  $E$ .

		A >= 20		12 <= A <= 19	
		(A >= 1013) or (A >= 360 and E >= 1088)	(A < 360) or (A < 1013 and E < 1088)	E - A <= 175	E - A > 175
Max polar code exponent	n <sub>max</sub>	10			
Input bits interleaver flag	I_IL	0			
Channel Interleaver flag	I_BIL	1			
Segmentation flag	I_seg	1	0		
CRC Length	L	11		6	
Number of PC bits	n <sub>pc</sub>	0		3	
Number of row-weight PC bits	n <sub>wm_pc</sub>	0		0	1

**FIGURE 2. Code parameters and bounds**

### 3.1.1. Set the flag variables and attach CRC

The values of the flag variables are set according to the payload length A and codeword length E as shown in Figure 2. If the segmentation flag is set, there exists two code blocks. CRC bits are calculated using a shift register. The value of each CRC bit is calculated by finding the mod 2 of the sum of the values in the CRC shift register and CRC polynomial. Attach the CRC bits to the payload. The length of each code block is K.

### 3.1.2. Calculation of n

The polar code length is given by  $N = 2^n$ .

$n = \max(\min(n_1, n_2, n_{max}), n_{min})$ ;  
 $n_{min} = 5$  and  $n_{max} = 10$  gives the lower and upper limit of polar code length.

$n_2$  is calculated as  $\text{ceil}(\log_2(8K))$  where  $n_2$  is the upper limit on the code depending on code rate (minimum).

$n_1$  that is obtained based on the below conditions, gives the limit for rate matching selection :

if  $(\log_2(E) - \text{floor}(\log_2(E))) < 0.17$  and  $K/E < 9/16$ ,  $n_1$  is calculated as  $\text{floor}(\log_2(E))$ , otherwise,  $n_1$  is calculated as  $\text{ceil}(\log_2(E))$ .

### 3.1.3. Creation of frozen and information set:

To create the frozen set, extract the indices smaller than N from the pre-defined universal reliability sequence. The first few bits contained in the frozen set refer to the bits eliminated from the codeword by rate matching schemes. The three steps in the formation of frozen set are:

i. Pre-freezing: This step is performed if  $K/E \leq 7/16$ . Sub-block interleaving is carried out through the length of the bits discarded from codeword by rate matching schemes. The N encoded bits are divided by the interleaver into 32 blocks (each of length  $N/32$  bits) based on a pre-defined sub-block interleaver pattern containing 32 integers. Hence,

the result obtained after sub-block interleaving is given by

$$\text{sub\_block\_interleaver\_pattern}((\text{floor}(32*n/N))+1)$$

$$*N/32 + \text{mod}(n, N/32)$$

This result is then appended to a temporary frozen list.

ii. Extra freezing: This step is performed if  $K/E \leq 7/16$  and  $E \geq 3N/4$ . To prevent the information set from becoming incapable due to puncturing, extra freezing is done.

iii. Reliability freezing: In reliability freezing, a temporary information set is obtained which contains the elements present in the universal reliability sequence (smaller than N) but not in the temporary frozen list. The final information set will comprise of the most reliable indices that contains the indices of the parity check bits and message bits.

The final frozen set will contain the indices present in the universal reliability sequence (smaller than N) that are not in the final information set.

### 3.1.4. Calculation of the indices of the PC bits

Calculate 'n-fold Kronecker product' and calculate the sum of each row to obtain the row weights.

To obtain most reliable indices in the information set, extract indices in the range - the no. of parity check bits and size of the information set. Consider row weights of those rows that are valid after most reliable indices in the information set. Find the minimum row weight.

If the no. of row weight parity check is set to 1, in the information set, it is stored in the indices that have minimum row weight. If there is more than one row having same minimum row weight, find the last occurrence of the minimum row weight and extract the corresponding most reliable index and add it to the parity check list.

If the no. of row weight parity check is set to 0, the indices of the parity check bits will only contain the first  $n_{pc}$  bits from the information set;  $n_{pc}$  is the no. of parity check bits.

**3.1.5. Calculation of PC bits:**

If no. of parity check bits is greater than 0, the PC bits are found using a ‘cyclic shift register’ (length 5-  $y_0, y_1, y_2, y_3, y_4$ ). Along the length of the payload, if the index of a particular bit is present in the information set and is present in the list containing the indices of the parity check bits, the value in  $y_0$  is added to a vector. If the index is not present in the list containing the indices of the parity check bits, the value in the code block is added to a vector.  $y_0$  is updated with the xor performed on the existing value of  $y_0$  and the value in the code block. If the value is neither present in the information set nor in the list containing the indices of the parity check bits, 0 is added to the vector. If  $n_{pc} \leq 0$ , the information bits are added into the vector while the remaining bits are 0. The final encoded block will be the mod2 of the product of the values in the vector and the generator matrix obtained after n-fold Kronecker product.

**3.1.6. Perform sub block interleaving.**

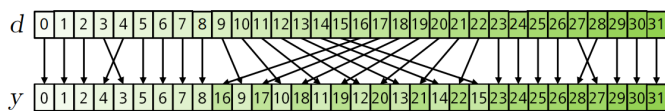


Fig. 10: Design of the sub-block interleaver J.

**FIGURE 3. Design of the sub block interleaver**

**3.1.7. Perform rate matching**

Possible schemes for rate matching are as follows:

If  $E \geq N$ , perform repetition wherein the encoded bits are repeated. Using repetition, bits are sent multiple times within the transmission.

If  $K/E \leq 7/16$ , perform puncturing wherein the bits are removed from the start of the sub block.

If  $K/E > 7/16$ , perform shortening wherein the bits are removed from the end of the sub block.

Check if the flag  $I_{BIL}$  is set to 1. If it is set to 1, calculate ‘T’ as  $T = \text{ceil}(\sqrt{(8 \cdot E) + 1} - 1) / 2$ . T is used in the construction of the interleaving matrix.

If  $I_{BIL}$  is not set to 1, the rate matched bits matrix will contain the bits obtained after rate matching.

**3.1.8. Modulation**

Modulate the received encoded bit vector using BPSK.

**Channel**

Construct a Rayleigh channel as the sum of two gaussian random variables.

**3.2. Decoder**

**3.2.1. Demodulation**

Demodulate the bits received from the noise channel using BPSK.

**3.2.2. De-concatenation and de-rate matching**

Based on the segmentation flag value (0/1), set the number of code blocks to (1/2) and divide the encoded bits into two parts if segmentation flag value is 1. Bits in each code block are rearranged according to the rate-matching scheme.

**3.2.3. SC Decoding**

The proposed SC decoding scheme can be summarised using the algorithm below.

- Check the length of the received vector consisting of demodulated bits
- If the length of the received vector is not 2 go to step 5
- If length of the received vector is 2, pass the two bits as the parameters of the f function which performs ‘minsum operation’.  $f(r_1, r_2) = \text{sgn}(r_1) \text{sgn}(r_2) \min(-r_1, -r_2)$  If the value returned by the f function  $\geq 0$  set  $u(1) = 0$  else set  $u(1) = 1$
- Pass the two bits in the received vector along with the decision  $u(1)$  to the g function.  $g(r_1, r_2, u(1)) = r_2 + (1 - 2(u(1)))r_1$ . If the value returned by the g function  $\geq 0$  set  $u(2) = 0$  else set  $u(2) = 1$  Then, Set  $v(1) = \text{bitxor}(u(1), u(2))$   $v(2) = u(2)$ . Return the vectors u and v.
- Pair the received vector in pairs of two. Pass each of the pairs  $(r_1, r_2)$  to the f function which performs ‘minsum operation’,

$$f(r_1, r_2) = \text{sgn}(r_1) \text{sgn}(r_2) \min(|r_1|, |r_2|)$$



- Repeat step 5 until the length of the vector becomes 2. When the length becomes 2, go to steps 3 and 4. Store the returned vectors as  $u_1$  and  $v_1$ .
- Pass each of the pairs to the  $g$  function along with their corresponding bit in vector  $v_1$ .

$$g(r1, r2, \text{corresponding bit in vector } v_1) = r2 + (1 - 2(\text{corresponding bit in vector } v_1))r1$$

- Repeat steps 2 to 7 until the length of the vector becomes 2. When the length becomes 2, go to steps 3 and 4. Store the returned vectors as  $u_2$  and  $v_2$ .
- Combine ( $u_1$  and  $u_2$ ) to form the final vector  $u$  and return it

#### 3.2.4. De-segmentation and calculation of bit error rate

Depending on the number of code blocks, the bits are concatenated to form one block after removal of CRC.

Total no. of wrongly decoded bits is to be found out and the bit error rate is calculated as follows:

$$\frac{\text{Number of erroneous bits}}{\text{Total number of bits}}$$

## 4. Results and discussions

The BER plot shows the variation of BER values of various payload sizes with  $E_b/N_0$  (in dB). The BER value gradually decreases as  $E_b/N_0$  value increases. It is observed that initially the BER values are small for lower payload size. For a payload size of 256, the BER plot starts from 0.449 then decreases to 0.387 and then further decreases to 0.012 and becomes 0 when  $E_b/N_0$  is 15 dB. For a payload size of 128, the BER plot starts from 0.445 then decreases to 0.179 and becomes 0 when  $E_b/N_0$  is 10 dB. For a payload size of 64, the BER plot starts from 0.328 then decreases to 0.078 and becomes 0 when  $E_b/N_0$  is 10 dB.

## 5. Conclusion

This paper presents the implementation of the transmit and receive chain of polar coding systems according to 5G standards. After generating the bit sequence, we start the process of polar encoding and SC Decoding algorithm was used for decoding the



FIGURE 4. BER Plot

bits. Repetition, puncturing, and shortening were used as the rate matching techniques and also  $crc6$  and  $crc11$  were used to evaluate the codes as defined in the '3GPP standard'. A BER graph was also plotted to visualize the block error rate for various payload sizes. 5G polar codes were observed to perform well with short block lengths.

## References

- Arıkan, Erdal. "Systematic Polar Coding". *IEEE Communications Letters* 15.8 (2011): 860–862.
- Bae, Jung Hyun. "hsien-ping lin, kee-bong song and jungwon lee, "An overview of channel coding for 5G NR cellular communications". *SIP* 8 (2019).
- Bioglio, Valerio, Carlo Condo, and Ingmar Land. "Design of Polar Codes in 5G New Radio". *IEEE Communications Surveys & Tutorials* 23.1 (2021): 29–40.
- Chen, Jie, et al. "Distributing CRC Bits to Aid Polar Decoding". *2017 IEEE Globecom Workshops (GC Wkshps)* (2017).
- Cheng, Fengyi, et al. "CRC-Aided Parity-Check Polar Coding". *IEEE Access* 7 (2019): 155574–155583.
- Condo, Carlo, Furkan Ercan, and Warren J Gross. "Improved successive cancellation flip decoding of polar codes based on error distribution". *2018 IEEE Wireless Communications and Networking Conference Workshops (WCNCW)* (2018).
- Kamenev, Mikhail, Yulia Kameneva, and Oleg Kurmaev. "Alexey Maevskiy, "Permutation Decoding of Polar Codes". *2019 XVI International Symposium "Problems of Redundancy in Information and Control Systems* (2019).

Maltiyar, Kaveri and Deepti Malviya. "Polar Code: An Advanced Encoding And Decoding Architecture For Next Generation 5G Applications". *International Journal on Recent and Innovation Trends in Computing and Communication* 7.5 (2019): 26–29.

Oliveira, Robert M and Rodrigo C De Lamare. "Design of Rate-Compatible Polar Codes Based on Non-Uniform Channel Polarization". *IEEE Access* 9 (2021): 41902–41912.

Tal, Ido and Alexander Vardy. "List Decoding of Polar Codes". *IEEE Transactions on Information Theory* 61.5 (2015): 2213–2226.

Trifonov, Peter and Vera Miloslavskaya. "Polar codes with dynamic frozen symbols and their decoding by directed search". *2013 IEEE Information Theory Workshop (ITW)* (2013).



© Prahlad D et al. 2023 Open Access. This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.

**Embargo period:** The article has no embargo period.

**To cite this Article:** , Prahlad D, Aishwarya Puthuraya , Gayathri Srinivasan , Harsh Kumar , and Keerthi N Reddy . "Polar Coding in 5G systems." *International Research Journal on Advanced Science Hub* 05.05 May (2023): 175–180. <http://dx.doi.org/10.47392/irjash.2023.033>