RESEARCH ARTICLE

RSP Science Hub

International Research Journal on Advanced Science Hub
2582-4376
www.rspsciencehub.com
Vol. 06, Issue 05 May

Check for updates

http://dx.doi.org/10.47392/IRJASH.2024.019

Effect of an Edge Crack on Static Deflection and Natural Frequency in a Composite Material Cantilever Beam

Aditi Poudwal[1], Sanskruti Panaskar[2], Pranjal Raul[3], Isha Lagad[4], Sunil Pansare[5]

[1,2,3,4,5]*Department of Mechanical Engineering, St. Francis Institute of Technology, Mumbai, India.*

Emails: *poudwal.aditi@gmail.com[1], sanskritipanaskar@gmail.com[2], raulpranjal112@gmail.com[3], ishalagad@gmail.com[4], sunilpansare@sfit.ac.in[5]*

Article history

Received: 26 April 2024
Accepted: 13 May 2024
Published: 30 May 2024

***Keywords**:*
Composite materials, Medium-Density Fibreboard (MDF), Static deflection, Inclined Edge crack, Natural Frequency, Structure Health Monitoring (SHM)

Abstract

The presence of cracks in composite materials poses a significant risk of catastrophic failure. Composites are used under diverse environmental conditions, and studying cracks helps estimate service life, ensuring durability. The composite material used in this project is MDF (Medium- Density Fibreboard). MDF is an engineered wood product known for its flexibility for curved surfaces, cost-effectiveness, and knot-free composition, making it ideal for furniture, cabinets, and wall panels. The main aim of this work is to study the effect of an inclined edge crack on static deflection and natural frequency in a composite material cantilever beam, using finite element analysis. This includes examining the effect of crack parameters like location, relative crack depth, and crack inclination angle on the beam's static deflection and natural frequencies of flexural vibrations. Studying static deflection provides insights into structural stability while exploring natural frequencies aids in predicting dynamic responses in a cracked cantilever beam

1. Introduction

Cracks in composite materials pose a significant risk of catastrophic failures if not addressed promptly. To maintain structural safety, understanding the behaviour of such cracks is essential. Composites are frequently subjected to a range of environmental conditions, including moisture, temperature fluctuations, and mechanical stress. Studying these cracks enables engineers and researchers to comprehend how com- posites deteriorate over time, which is vital for estimating their service life and ensuring durability. Understanding the impact of cracks on composite materials facilitates informed decisions regarding material selection for specific applications. Engineers can select composites optimally suited to endure the expected stressors and environmental conditions. Further- more, research and development initiatives aimed at creating new composite materials with enhanced fracture resistance and other beneficial properties are advanced by an improved understanding of cracks in composites.

2. Problem Statement

When a component develops a crack, its stiffness reduces locally. This alteration in local stiffness directly impacts the component's dynamic and static behaviour. Parameters such as static deflection, natural frequencies, modes, damping co-efficient, etc. change. Studying static deflection in composite materials is essential for understanding its behaviour and performance in various applications. It assists engineers in ensuring that structures can withstand loads without excessive deformation, thereby guaranteeing safety and functionality. Modal analysis, which identifies natural frequencies and mode shapes, is crucial for predicting dynamic behaviour and ensuring reliability across various sectors such as aerospace, automotive, civil engineering, and manufacturing.

3. Objectives

The primary objective of this study is to measure and characterize the static deflection behaviour of the cantilever beam with and without the edge crack under the same load. This will provide insights into how the presence of the crack affects the static deflection of the beam. Additionally, this investigation seeks to determine the first 4 natural frequencies of transverse vibrations through modal analysis, aiming to understand its dynamic behaviour to utilize modes obtained through modal analysis to visualize and comprehend the dynamic deformation patterns of transverse vibrations.

4. Literature Review

4.1 Causes of Cracks in Composite Materials

Composite materials can develop cracks due to various factors such as mechanical overloading, stress concentrations, environmental exposure, manufacturing defects, and inter- laminar shear stresses. mechanical overloading and stress concentrations initiate and propagate cracks, requiring careful management. Environmental factors like moisture, chemicals, and temperature variations can deteriorate materials, leading to fractures. Production errors such as voids and resin in- consistencies act as stress raisers. High-energy impacts and differential loading in layered composites also induce cracks and delamination. Prolonged exposure to constant pressures can cause material deformation and microscopic crack initi- ation. Understanding and managing these factors are crucial for preventing crack formation and ensuring the integrity of composite materials. [1,2]

4.2 Types of Cracks in Composite Materials

Cracks in composite materials can originate from manufac- turing discrepancies, environmental adversities, and external forces. matrix cracking, triggered by thermal anomalies or cyclic loads, diminishes stiffness and strength. Fiber breakage, due to excessive tensile forces, can lead to catastrophic fail- ure under significant loads. Delamination, induced by shear, bending, or impact forces, and interlinear cracking, occur- ring between adjacent plies—especially in areas of geometric discontinuities—are of particular concern. Micro cracking, caused due to external stressors or thermal cycling, undermines material integrity. Furthermore, exposure to severe conditions exacerbates composite degradation, affecting both the matrix and fibres, thereby facilitating fracture development. The emergence of edge cracks and voids further concentrates stress, emphasizing the importance of a thorough understanding of these crack types for design using composite materials. [3-5]

4.3 Medium-Density Fiberboard (MDF)

Medium-density fibreboard (MDF) is an engineered wood product made from wood fibres, wax, and resin binder, offering advantages such as flexibility, stability, and cost-effectiveness. It finds diverse applications in furniture, construction, and interior design due to its superior resistance to heat and humidity. MDF faces various structural issues including surface cracks from rapid drying and mechanical stress, edge cracks during cutting, splitting due to moisture, and internal delamination from moisture penetration. [6-9] Properties of MDF is shown in Table 1.

Table 1 Properties of MDF [8]

S.No	Properties	Value
1.	Young's Modulus (E)	4 GPa
2.	Poisson's ratio (μ)	0.25
3.	Density (ρ)	500-800 kg/m^3

4.4 Timoshenko and Euler Bernoulli Beam

The Euler-Bernoulli Beam Theory is based on the assumption of very small shear deformations and also negligible effect of the Poisson ratio. It is best suited for analysing long and slender beams. In contrast, the Timoshenko Beam Theory incorporates shear deformation and cross-section rotation and offers a more accurate depiction of short beams [10]. The applicability of these theories is determined by the beam's aspect ratio. It is the ratio of beam length (L) to depth of the cross-section. The Euler-Bernoulli theory is more suitable to aspect ratios greater than 10, and the Timoshenko theory for ratios less than 10 [11]. Euler and Timoshenko Beam After Applying Load image is shown in Figure 1.

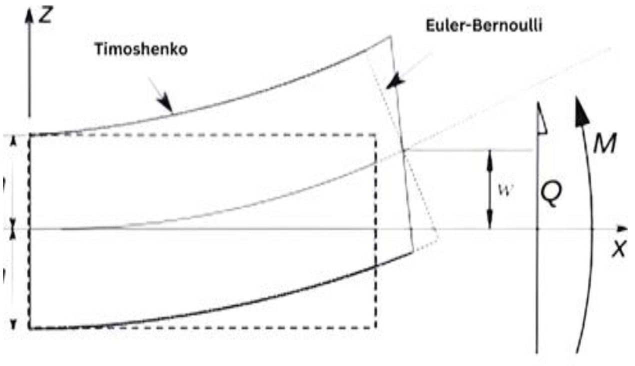

Figure 1 **Euler and Timoshenko Beam After Applying Load**

4.5 Importance of Static deflection and Natural Frequency

Static deflection, in the context of structural engineering and materials science, refers to the amount of deformation or displacement that a structure or material experiences when subjected to a static load or force. Static deflection analysis provides valuable insights into how composite materials be- have under constant loads. Excessive deflection can lead to structural instability, which may compromise safety. Under- standing the material's behaviour under static loads is vital for ensuring long-term durability. When a system experiences an initial disturbance and subsequently vibrates on its own, the frequency at which it oscillates without any external excitation is referred to as its Natural frequency. As the system vibrates at these natural frequencies, specific patterns emerge, known as mode shapes or modes, determined by the directions in which different components deflect. Modal analysis, a method employed in understanding the dynamic characteristics of mechanical systems, focuses on identifying natural frequencies, damping ratios, and modes. This technique is instrumental in discerning potential sources of vibration, forecasting structural behaviour under dynamic loads, and optimizing designs to avert resonance. [13]

4.6 Literature Review

Many researchers have worked in the field of crack detection using changes in static deflection and natural frequencies. Some of the studies are reviewed in the following paragraphs, for the purpose of throwing some light on the study at hand: Caddemi and Morassi [14] developed a method to detect a single crack in a beam by studying changes in static deflection resulting from damage. They modelled the crack using a linear spring that connects two segments of the beam adjacent to the crack. The authors concluded that the accuracy of their theoretical findings was well-supported by static measurements slender beams. In contrast, the Timoshenko Beam Theory incorporates shear deformation and cross-section rotation and offers a more accurate depiction of short beams [10]. The applicability of these theories is determined by the beam's aspect ratio. It is the ratio of beam length (L) to depth of the cross-section. The Euler-Bernoulli theory is more suitable to aspect ratios greater than 10, and the Timoshenko theory for ratios less than 10 [11]. from steel beams containing cracks, and they discussed the potential for extending their analysis to detect multiple cracks in beams. Naik [15] introduced a technique for detecting a single- edge normal crack in pipes, involving measurements of static deflections at two points on a slender beam, configured in both cantilever and simply supported setups. Pansare and Naik [16] proposed a method based on static deflection to detect inclined edge cracks in prismatic cantilever beams. They used a rotational spring to represent the crack's flexibility at the crack tip and conducted experiments on twenty-one mild steel specimens, varying in crack inclinations, locations, and depths. Ramesh et al. [17] aimed to establish a relationship between modal natural frequencies and crack depths in aluminium cantilever beams. The authors used vibration analysis con- ducted through ANSYS software, validated by experimental tests using a universal vibration apparatus, highlighting how modal frequencies correlate with crack depths. Mia et al. [18] studied the relationship between modal natural frequencies and crack depths in aluminium cantilever beams, enhancing understanding of how crack depth affects the dynamic properties of structures. Khalkar and Ramachandran [19] explored the stiffness of cracked cantilever beams with varying depths and geometries using deflection and vibration methods. They concluded that relationships between cracked cross-sections, depths, locations, and stiffness are significant, with natural frequencies computed for spring steel materials EN 8 and EN 47 showing higher accuracy for U-shaped and rectangular-shaped cracks compared to V-shaped ones. Bhagat and Kadhane [20] focused on modelling rectangular and angular edge cracks in an aluminium cantilever beam. They employed a combination of finite element analysis and experimental techniques, using accelerometers and impact hammers to measure the beam's response and frequencies, re- sportively, analysed using an oscilloscope. The study provided insights into how crack shape, location, and depth affect the natural frequency of the beam, employing both experimental and numerical approaches to comprehensively investigate this phenomenon. The literature review explores methods for detecting cracks in structural elements such as beams and pipes using static deflection and frequency analysis. The crack is represented using linear springs or rotational springs. Experimental validations demonstrate the effectiveness of these methods, with promising results in accurately predicting crack locations and severity. However, challenges such as crack detection accuracy and theoretical model refinement remain areas for further research. Overall, the studies underscore the significance of static deflection and frequency analysis in non-destructive evaluation techniques for detecting and assessing structural damage. In this study, the main aim is to provide insight into the effect of various crack parameters, viz relative crack depth, location and inclination on the static deflection and first four natural frequencies of transverse vibrations. This study attempts to investigate the impact of inclined edge crack on composite materials. Specifically, this study seeks to elucidate the relationships between crack parameters and both, static deflection and natural frequencies of flexural vibrations in Medium-Density Fibreboard (MDF), long and short, cantilever beams, employing Finite Element Analysis (FEA).

5. Methodology

The study commenced with the formulation of a problem statement, aimed at identifying the specific issue to be addressed (Figure 2). A comprehensive literature review was conducted to gain an understanding of the existing knowledge in the field of static deflection. Following this, a market survey was undertaken to know the cost and availability of various composite materials, leading to the selection of Medium- Density Fibreboard (MDF) for the study. ANSYS software, version 2020 R2, was chosen for Finite Element Analysis (FEA). The dimensions of the cantilever beam were decided by the available cross-sections in the market aided by Timoshenko and Euler-Bernoulli beam theories for the length. Parameters defining the crack, such as location, relative crack depth, and inclination angle, were specified in order to have a comprehensive study. A digital representation of the beam was created through CAD modelling. Analysis was performed using ANSYS Workbench 2020 R2. Results were then represented using various graphs.

6. Geometric Modeling

The aspect ratio of the beam is typically defined as the ratio of its length (L) to its cross-sectional dimension depth (h). If the aspect ratio is greater than 10 then the Euler-Bernoulli beam model is suitable and if it is less than 10 the Timoshenko beam model is to be used. (Refer Tables 2 and 3)

Table 2 Dimension Table for Timoshenko and Euler- Bernoulli Beams

Parameter	Timoshenko Beam	Euler-Bernoulli Beam
h (mm)	32	32
L = 9*h (mm)	288	480
Width (mm)	19	19

Table 3 Crack Parameters for Timoshenko and Euler- Bernoulli Beams

Location (β)	0.2, 0.4, 0.6, 0.8
Relative Crack Depth (a)	0.2*h, 0.3*h, 0.4*h, 0.5*h
Crack Inclination (θ)	30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 150°

Table 4 Various Cases of Timoshenko Beam

a	6.5 mm	9.5 mm	13 mm	16 mm
a/h	0.2	0.3	0.4	0.5
β	57.6 mm	115.2 mm	172.8 mm	230.4 mm
β/L	0.2	0.4	0.6	0.8

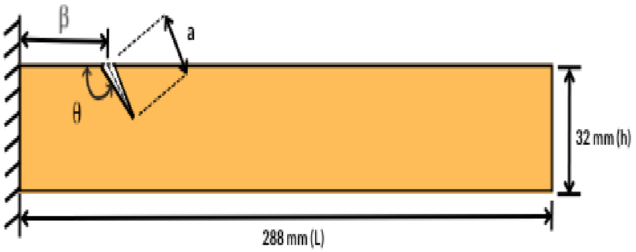

Figure 2 Timoshenko Beam Model with Crack Parameters

Table 5 Various Cases for Euler-Bernoulli Beam Cases

a	6.5 mm	9.5 mm	13 mm	16 mm
a/h	0.2	0.3	0.4	0.5
β	96 mm	192 mm	288 mm	384 mm
β/L	0.2	0.4	0.6	0.8

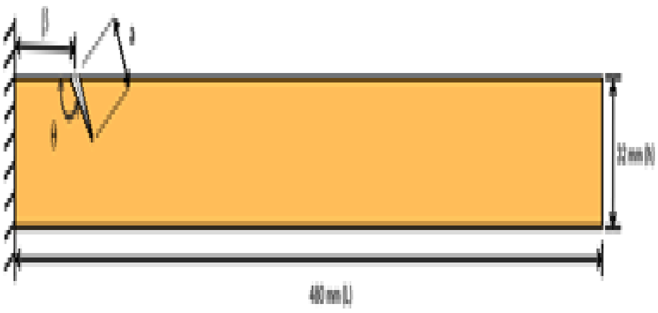

Figure 3 Euler-Bernoulli Beam Model with Crack Parameters

Tables 4 and Table 5 provide a detailed overview of parameters for both the Timoshenko and Euler-Bernoulli beams, facilitating reference for varying crack depths and locations. The material selected for this study is Medium- density fibreboard (MDF). Two distinct beam models, the Timoshenko beam and the Euler-Bernoulli beam, are used, with lengths of 288 mm and 480 mm respectively. Both models consider a width of 19 mm and a depth of 32 mm. The investigation includes a range of inclination angles from 30° to 150° (in steps of 15° each), alongside various crack depths from 20% to 50% (in steps of 10% each), and crack locations from 20% to 80% (in steps of 20% each) of the total beam length. This approach yields a total of 144 specimens (9 inclinations, 4 depths and 4 locations) for each beam type, resulting in 288 cases for finite element analysis (FEA) simulation (Refer Figure 3,4 & 5)

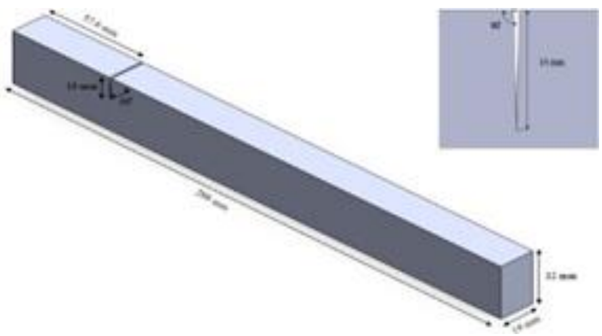

Figure 4 Solid Model of Timoshenko Beam with Crack for Location 1 (β =0.2, θ= 90, a/h = 0.5)

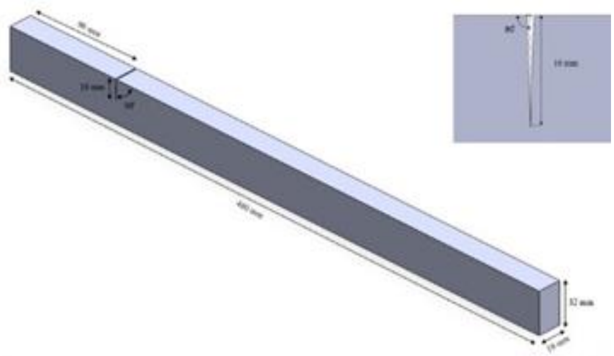

Figure 5 Solid Model of Euler-Bernoulli Beam with Crack for Location 1 (β =0.2, θ= 90, a/h = 0.5)

6.1 Static Deflection Analysis

The analysis was aimed at observing the deflection under a specified load condition for both Timoshenko and Euler- Bernoulli beams, with a particular focus on variations due to the presence of cracks. A point load of 100 N was applied in the negative Y direction, 15 mm from the free end of each beam type, and deflections were measured at the point of load application. The findings are summarized in Table 6 (Refer Figure 6 & 7).

Table 6 Results for Static Deflections

Beam type	Condition	Length, mm	Deflection-range, mm
Timoshenko	Untracked	400	3.2888
Timoshenko	Cracked	400	3.2944 to 5.7237
Euler-Bernoulli	Untracked	288	16.165
Euler-Bernoulli	Cracked	288	16.175 to 23.274

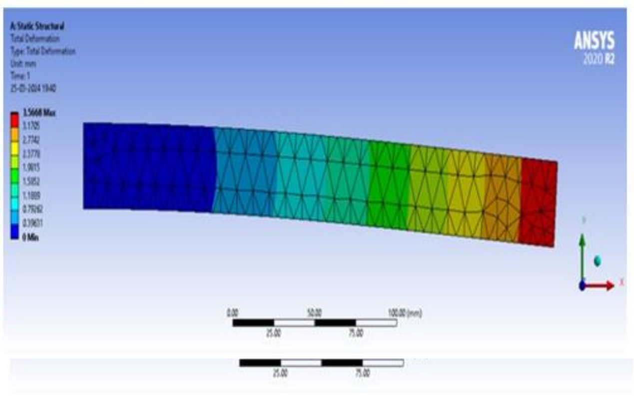

Figure 6 Static Deflection Obtained for Timoshenko Beam with- Out Crack

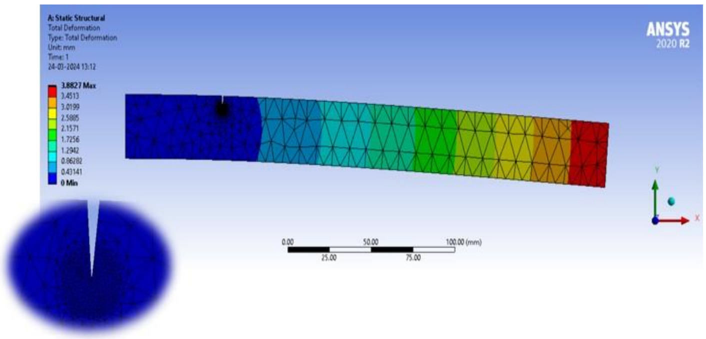

Figure 7 Static Deflection Obtained for Timoshenko Beam with Crack

6.2 Modal Analysis

Modal analysis was conducted for all beam combinations, with one end of the beam fixed. The analysis aimed to determine the natural frequencies for the first four transverse bending mode shapes. The study compared the behaviours of Timoshenko and Euler-Bernoulli beams, noting differences in frequency response due to beam length and bending mode. This difference is primarily because the frequency of a beam is inversely proportional to its length—the longer the beam, the lower its natural frequency (Refer 8 & 11).

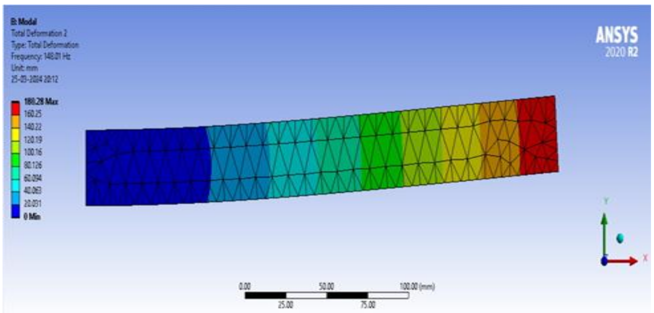

Figure 8 Mode 1 of Flexural Vibrations

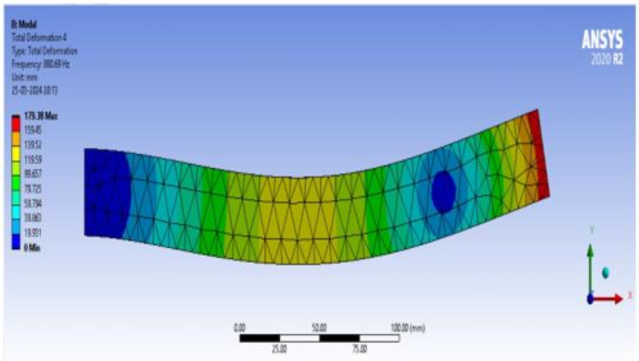

Figure 9 Mode 2 of Flexural Vibrations

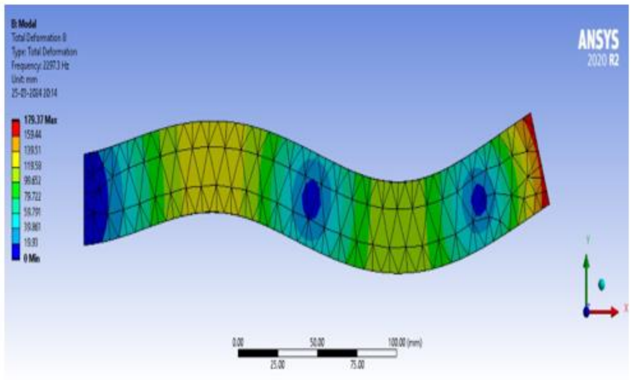

Figure 10 Mode 3 of Flexural Vibrations

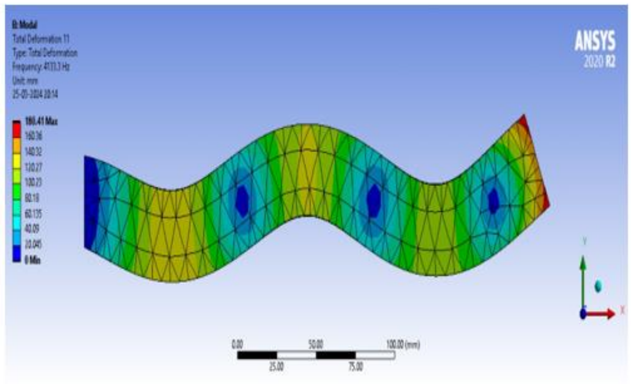

Figure 11 Mode 4 of Flexural Vibrations

7. Results and Discussion

7.1 Scatter plots for Static Deflection in Timoshenko and Euler Bernoulli Beams

Figure 12 illustrates the impact of crack location (β) on the static deflection of a Timoshenko beam. The deflection decreases as the crack moves from the fixed end (maximum at Location 1, β = 0.2) to the free end (minimum at Location 4, β =0.8), due to the higher bending moment at the fixed end. Similarly, Figure 13 demonstrates that a Euler-Bernoulli beam exhibits the similar pattern of deflection changes with crack location.

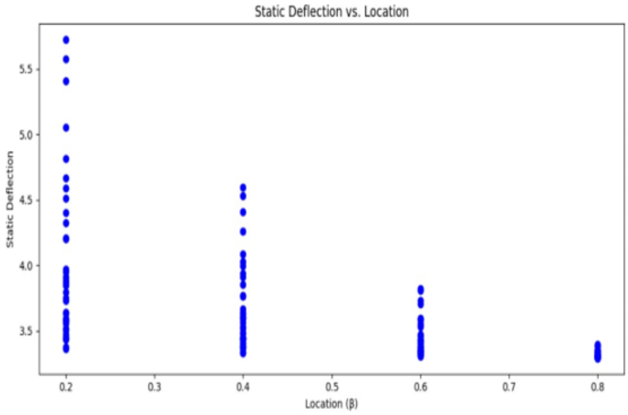

Figure 12 Static Deflection Vs Location for Timoshenko Beam

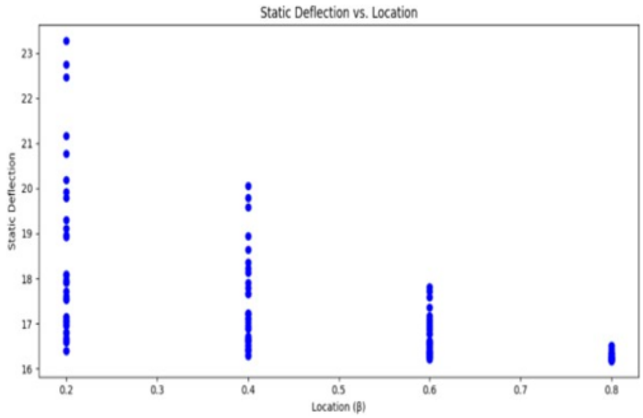

Figure 13 Static Deflection Vs Location for Euler BernoulliBeam

Figure 14 illustrates how the crack inclination angle (θ) affects the static deflection of a Timoshenko beam. Deflection increases from 30°, peaking at 90° where the crack is widest, then decreases towards 150° where the crack opening is minimal. Similarly, Figure 15 shows that the static deflection in a Euler-Bernoulli beam follows a similar pattern.

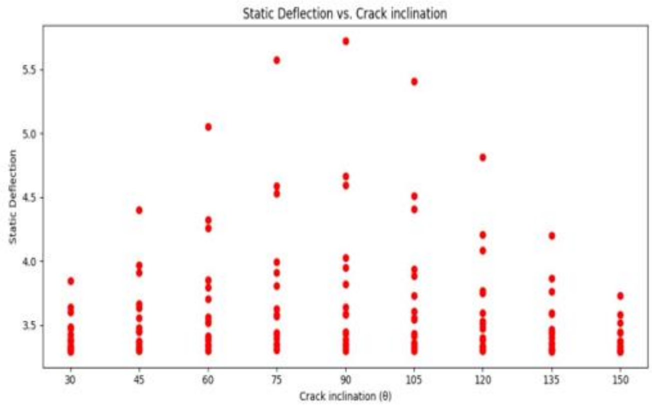

Figure 14 Static Deflection Vs Crack Inclination for TimoshenkoBeam

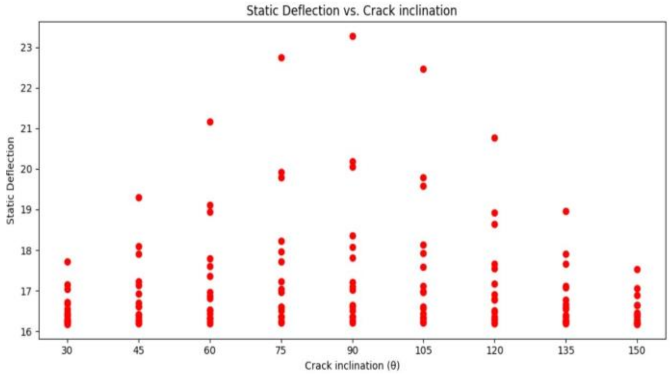

Figure 15 Static Deflection Vs Crack Inclination for Euler Bernoulli Beam

Figure 16 demonstrates how increasing relative crack depth (a/h) impacts the static deflection of a Timoshenko beam. The deflection increases with relative crack depth, being minimal at 6.5 mm (a/h = 0.2) and maximal at 16 mm (a/h = 0.5). This increase in deflection is due to the decreased effective cross-sectional area and the resulting reduction in beam stiffness as the crack deepens, compromising structural integrity and increasing deflection under load. Similarly, Figure 17 shows that a Euler-Bernoulli beam exhibits analogous behaviour concerning the effects of crack depth on static deflection.

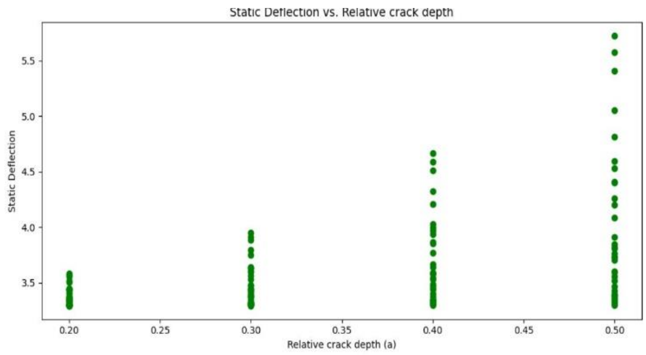

Figure 16 Static Deflection Vs Relative Crack Depth for Timoshenko Beam

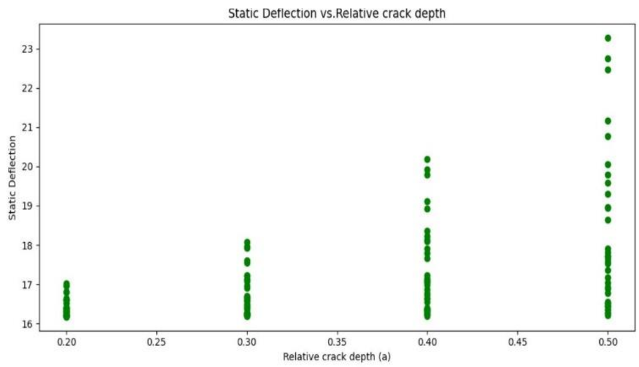

Figure 17 Static Deflection Vs Relative Crack Depth for Euler Bernoulli Beam

7.2 Interaction plots for Static Deflection in Timoshenko and Euler Bernoulli Beams at each location

Figures 18 and 19 illustrate the interaction between relative crack depth (a/h) and angle (θ) on the deflection at a specific location 1 (β = 0.2) of the Timoshenko and Euler-Bernoulli beams, respectively. Both figures demonstrate that as relative crack depth increases, static deflection also increases, with the minimum deflection occurring at a relative crack depth of 6.5 mm (a/h = 0.2) and the maximum at 16 mm (a/h = 0.5). Additionally, they show a decreasing trend in deflection with increasing crack inclination angle, following the sequence of 90°, 75°, 105°, 60°, 120°, 45°, 135°, 30°, and 150°. The maximum deflection occurs at an angle of 90°, corresponding to the maximum crack opening. Deflection for angles 105°, 120°, 135°, and 150° is slightly lower compared to angles 75°, 60°, 45°, and 30°, a discrepancy arising because the latter angles are closer to the fixed end of the beam, where the crack opening due to applied load is slightly greater. Conversely, angles 105°, 120°, 135°, and 150° are nearer to the free end of the cantilever beam. The bending moment at the fixed end is maximum resulting in the highest deflection. The maximum deflection recorded for location 1 (β = 0.2) is 5.7237 mm for the Timoshenko beam, and 23.274 mm for the Euler-Bernoulli beam, highlighting the differences in beam response due to structural design variations.

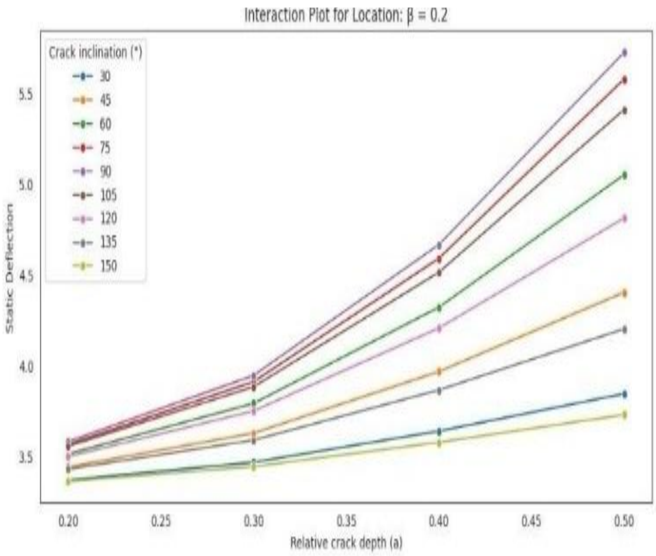

Figure 18 **Interaction plots for Location: β = 0.2 in Timoshenko Beam**

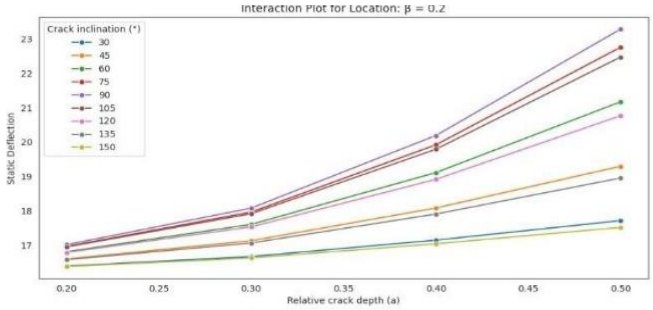

Figure 19 **Interaction plots for Location: β = 0.2 in Euler Bernoulli Beam**

Figures 20 (for Timoshenko beam) and 21 (for Euler- Bernoulli beam) display the effects of relative crack depth and inclination angle on deflection at location 2 (β = 0.4) of the beams. The maximum static deflection for the Timoshenko beam is 4.5959 mm, while for the Euler-Bernoulli beam, it is 20.062 mm, illustrating distinct structural responses between the two beam types.

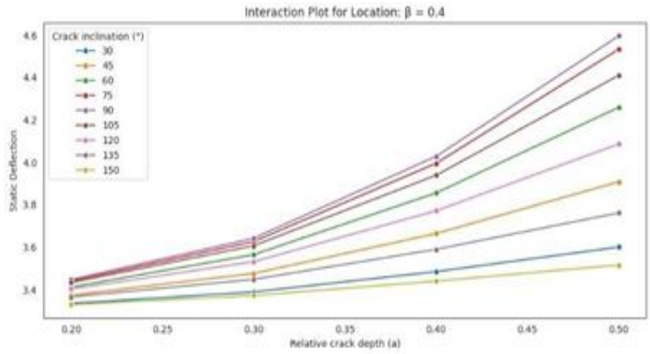

Figure 20 **Interaction plots for Location: β = 0.4 in Timoshenko Beam**

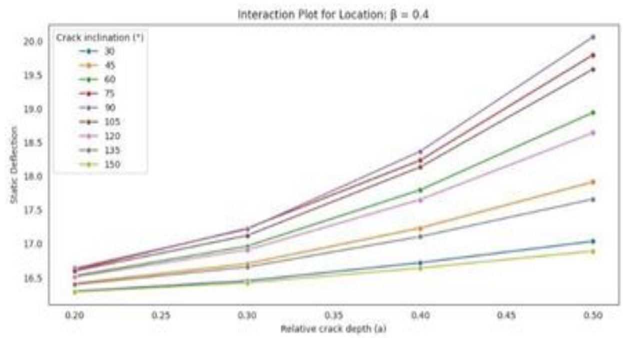

Figure 21 **Interaction plots for Location: β = 0.4 in Euler Bernoulli Beam**

Figures 22 and 23 illustrate the interaction between relative crack depth and crack inclination angle on the static deflection at location 3 (β = 0.6) of Timoshenko and Euler-Bernoulli beams respectively. Remarkably, both types of beams show an identical pattern for deflection with a maximum deflection of 3.8168 mm for the Timoshenko Beam and 17.803 mm for the Euler-Bernoulli Beam at location 3 (β = 0.6) respectively indicating a similar structural response to the specified conditions.

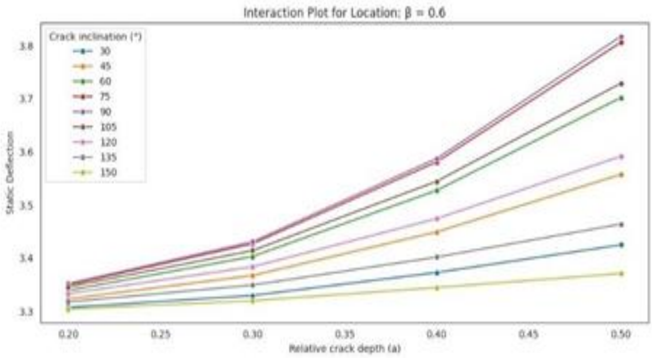

Figure 22 Interaction plots for Location: β = 0.6 in Timoshenko Beam

Figures 24 and 25 illustrate the interaction between crack depth and angle on the static deflection at location 4 (β = 0.8) for Timoshenko and Euler-Bernoulli beams respectively. Similarly, at location 4 (β = 0.8) both beams show an identical pattern for deflection with a maximum deflection of 3.82 mm for the Timoshenko Beam and 16.51 mm for the Euler-Bernoulli Beam respectively.

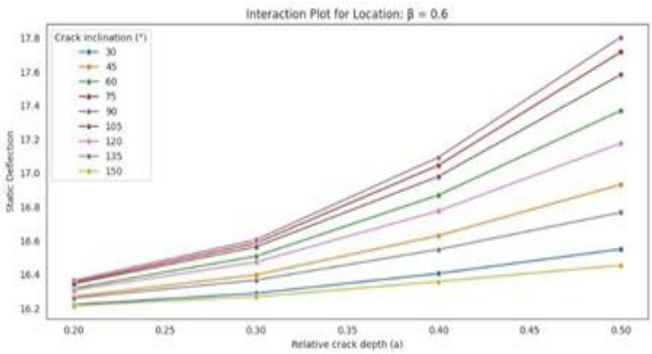

Figure 23 Interaction plots for Location: β = 0.6 in Euler Bernoulli Beam

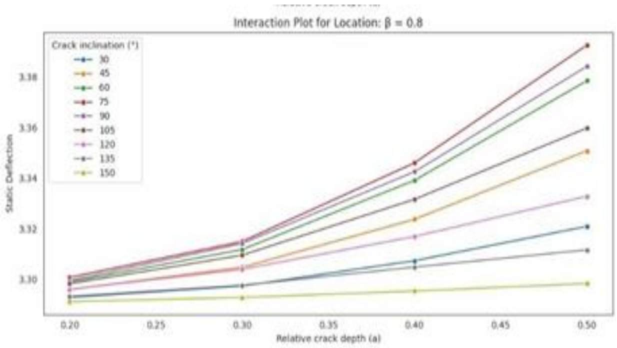

Figure 24 Interaction plots for Location: β = 0.8 in Timoshenko Beam

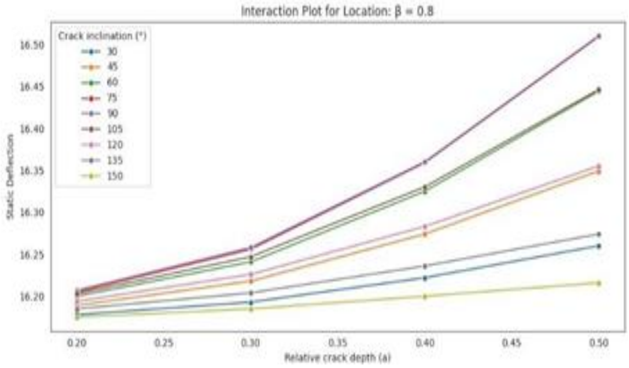

Figure 25 Interaction plots for Location: β = 0.8 in Euler Bernoulli Beam

7.3 Heat Map for Correlation of Different Variables for Static Deflection in Timoshenko and Euler Bernoulli Beam

To understand the impacts of location, crack inclination angle, and relative crack depth on the deflection of structural beams, Heat Map analyses were applied to both Timoshenko and Euler-Bernoulli beam theories. Figures 26 and 27 illustrate these relationships through correlation coefficients, providing insights into how these factors influence beam deflection under different theoretical frameworks for Timoshenko and Euler- Bernoulli Beam respectively.

7.3.1 Findings

- **Static Deflection vs. Location:** Both beams exhibit a moderate negative correlation (Timoshenko: 0.58, Euler-Bernoulli: 0.57), indicating that location has the least effect on static deflection as compared to relative crack depth and crack inclination angle. The static de- flection increases as the location approaches the fixed support.
- **Static Deflection vs. Crack Inclination:** A weak negative correlation is observed in both models (Timoshenko: 0.04, Euler-Bernoulli: 0.03), suggesting that an in- crease in the inclination angle of the crack slightly enhances static deflection. However, this influence is minimal, indicating that crack inclination is not a dominant factor in beam static deflection under the conditions studied.
- **Static Deflection vs. Relative Crack Depth:** A moderate positive correlation is noted (Timoshenko: 0.47, Euler- Bernoulli: 0.48), demonstrating that deeper cracks lead to greater static deflection. This correlation is consistent with the reduction in effective cross-

sectional area, which decreases the overall stiffness of the beam.

7.3.2 Discussion

The Heat Map effectively quantifies how static deflection is influenced by location, inclination angle, and relative crack depth within a structure or material.

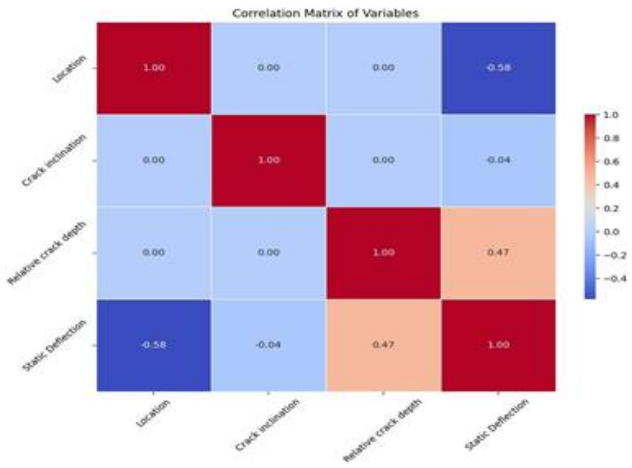

Figure 26 **Heat Map for Correlation of different variables for Static Deflection in Timoshenko Beam**

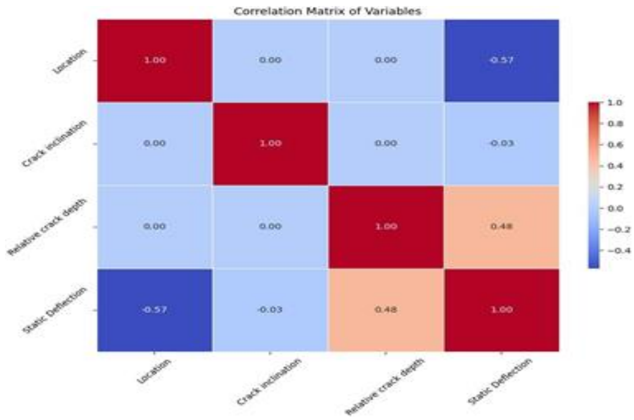

Figure 27 **Heat Map for Correlation of different variables for Static Deflection in Euler Bernoulli Beam**

7.4 Interaction plots for Natural Frequencies in Timoshenko Beams for each Mode shape (f1, f2, f3, f4)

Figures 28 and 29 show a clear pattern in the first two modes (f1 and f2) of Timoshenko beams. As the depth of the crack increases, the frequencies of these modes consistently decrease, regardless of the crack's inclination or location. This shows that there's a considerable decrease in frequency as the crack becomes deeper.

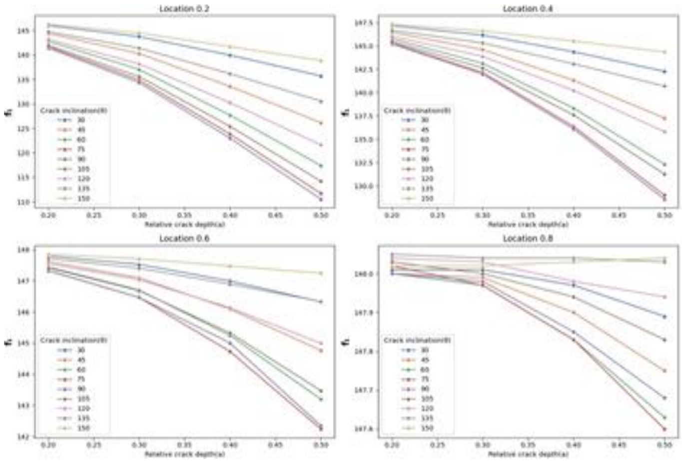

Figure 28 **Interaction plot for f1 at all Locations (β) in Timoshenko Beam**

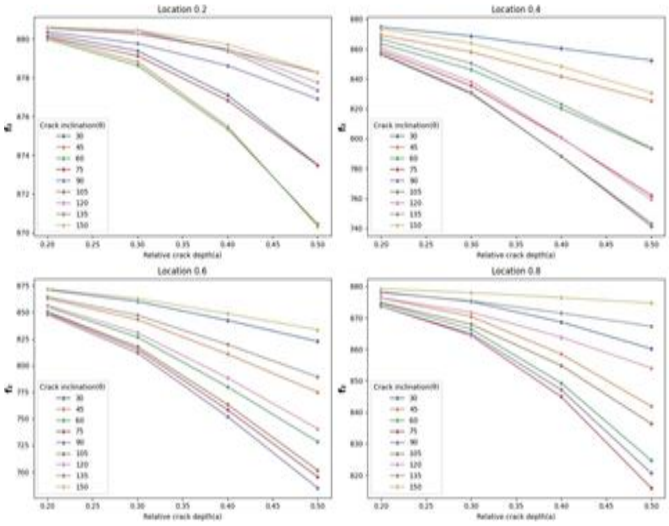

Figure 29 **Interaction plot for f2 at all Locations (β) in Timoshenko Beam**

Figure 30 illustrates a notable pattern in the third mode (f3) of Timoshenko beams at crack inclinations of 120°, 135°, and 150°. It shows that the frequencies decrease more significantly when cracks are closer to the beam's fixed end and as the relative crack depth increases. This trend suggests that both the proximity to the fixed end and the extent of the crack depth have a considerable impact on the natural frequencies. Figure 31 illustrates how the frequency response of fourth mode f4 changes with increasing relative crack depth. While the overall trend shows a decrease in frequency, the pattern varies with different crack inclinations. This variation points to a complex relationship between the relative crack depth and frequency, making predictions for mode f4 less consistent.

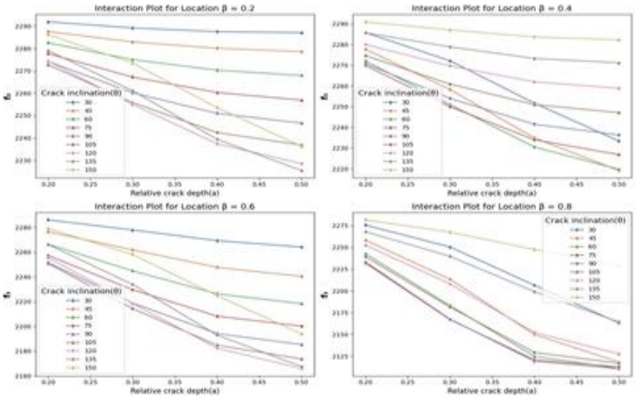

Figure 30 Interaction plot for f3 at all Locations (β) in Timoshenko Beam

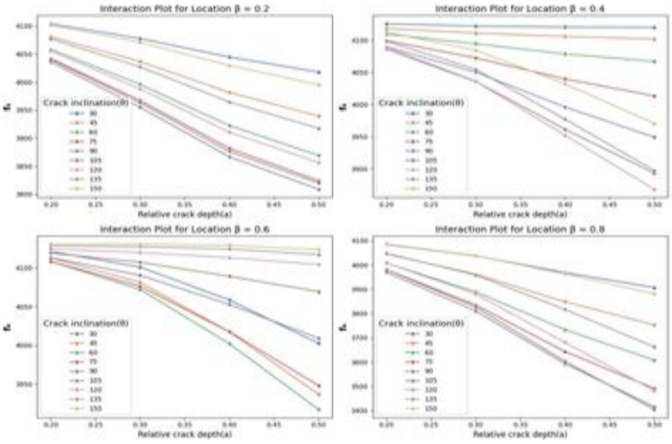

Figure 31 Interaction plot for f4 Beam at all Locations (β) in Timoshenko

7.5 Interaction plots for Natural Frequencies in Euler Bernoulli Beams for each Mode shape (f1, f2, f3, f4)

Figures 32-34 demonstrate the frequency response of Euler Bernoulli beams across modes f1, f2, and f3, showcasing a consistent pattern of frequency reduction as the relative crack depth increases. This consistency highlights the predictable impact of relative crack depth on frequency within these modes.

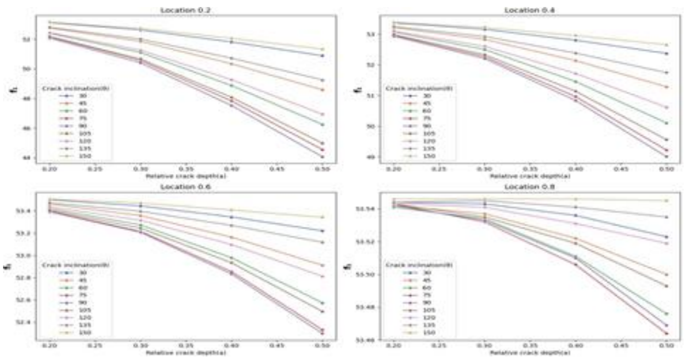

Figure 32 Interaction plot for f1 at all Locations (β) in Euler Bernoulli Beam

Figure 35 illustrates the behaviour of mode f4 in Euler- Bernoulli beams, revealing a consistent trend of frequency reduction as relative crack depth increases. Similar to the observations in Timoshenko beams, variations in the pattern across different crack inclinations are noted, suggesting complex interactions that affect the predictability of frequency changes.

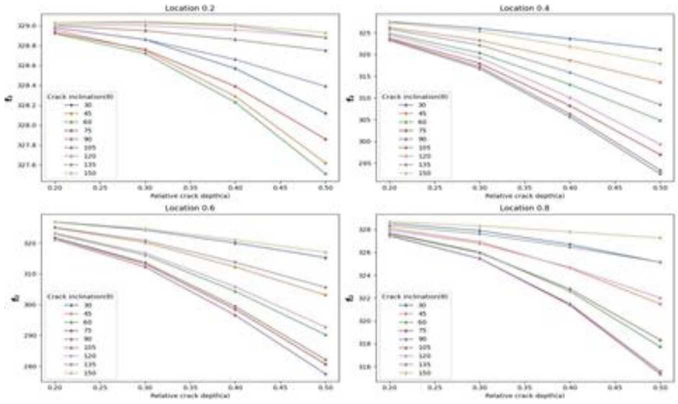

Figure 33 Interaction plot for f2 at all Locations (β) in Euler Bernoulli Beam

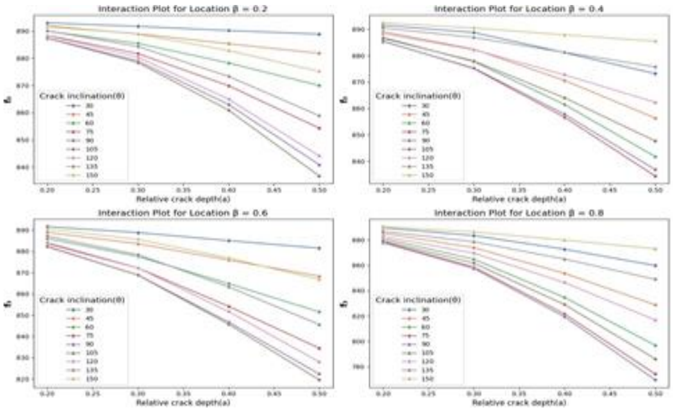

Figure 34 Interaction plot for f3 at all Locations (β) in Euler Bernoulli Beam

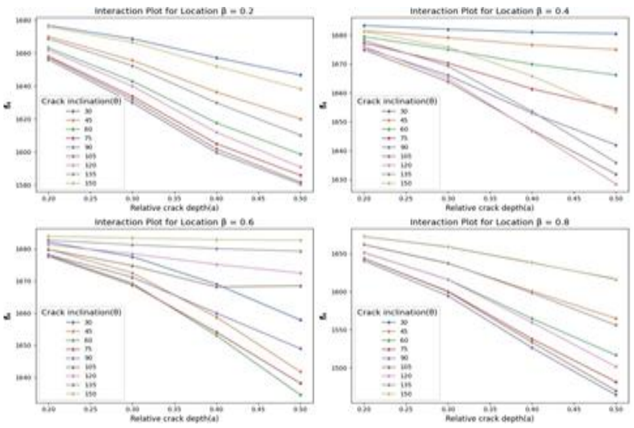

Figure 35 Interaction plot for f4 at all Locations (β) in Euler Bernoulli Beam

7.6 Heat Map for Correlation of different variables for frequencies in Timoshenko and Euler Bernoulli Beams (f1, f2, f3, f4)

This section explores the correlations between flexural vibration frequencies (f1, f2, f3, f4) and three key factors: crack location, crack inclination, and relative crack depth across both Timoshenko and Euler-Bernoulli beam theories. The analyses are visualized through Heat Maps, which elucidate the influence of these factors on the frequency behaviours of the beams (Refer 36 & 37).

7.6.1 Correlation between Frequencies and Crack Location

- **f1 vs. Location:** Exhibits a strong positive correlation in both beam models, indicating higher location values significantly increase f1.
- **f2 vs. Location:** Shows a weak negative correlation, suggesting a slight decrease in f2 as location values rise.
- **f3 vs. Location:** Reveals a moderate to strong negative correlation, indicating higher location values tend to reduce f3.
- **f4 vs. Location**: Demonstrates a weak to moderate negative correlation, with frequency f4 slightly decreasing as location values increase.

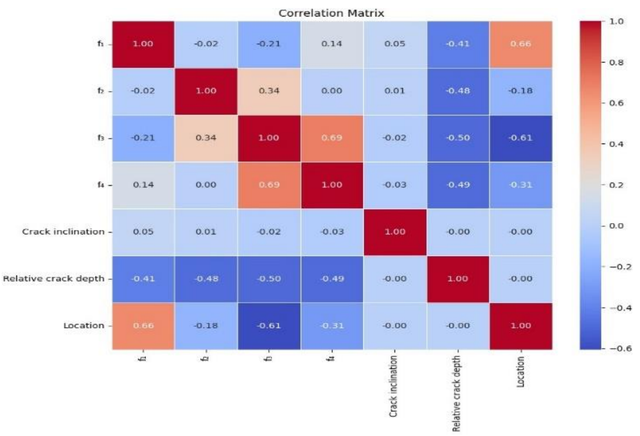

Figure 36 **Heat Map for Correlation of different variables for Natural Frequencies (f1, f2, f3, f4) in Timoshenko Beam**

7.6.2 Correlation between Frequencies and Relative Crack Depth

- **f1 vs. Relative Crack Depth:** Indicates a moderate negative correlation, suggesting that deeper cracks are associated with lower f1.
- **f2 vs. Relative Crack Depth:** Shows a moderate negative correlation, with a consistent decrease in f2 as crack depth increases.
- **f3 vs. Relative Crack Depth:** Reveals a moderate to strong negative correlation, indicating a significant reduction in f3 as depth increases.
- **f4 vs. Relative Crack Depth:** Displays a moderate negative correlation, with deeper areas experiencing a consistent reduction in f4.

7.6.3 Correlation between Frequencies and Crack Inclination

- **f1 vs. Crack Inclination:** Exhibits a very weak positive correlation, indicating minimal influence of angle on f1.
- **f2 vs. Crack Inclination:** Shows a very weak positive correlation, with changes in angle having minimal impact on f2.
- **f3 vs. Crack Inclination:** Reveals a very weak negative correlation, hinting that larger angles might slightly decrease f3.
- **f4 vs. Crack Inclination:** Demonstrates a very weak negative correlation, indicating a slight decrease in f4 with increasing angles.

8. Discussion

The analysis across both beam theories highlights that location and depth have significant effects on the frequencies, with location enhancing f1 but generally diminishing other frequencies, and depth consistently reducing all frequencies. The relative crack depth has minimal impact, suggesting that angular variations do not notably influence the vibrational characteristics of the beams under study.

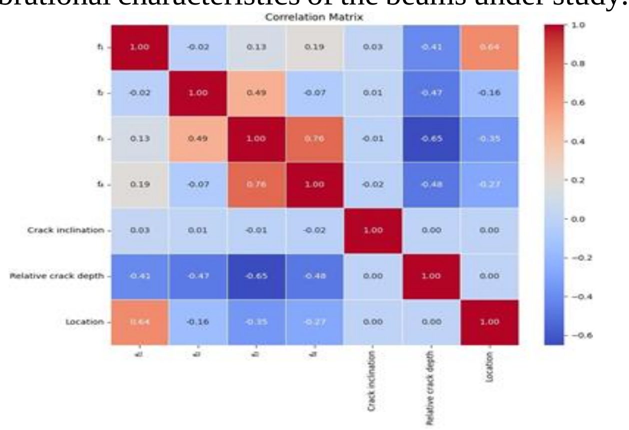

Figure 37 **Heat Map for Correlation of different variables for Natural Frequencies (f1, f2, f3, f4) in Euler-Bernoulli Beam**

Conclusions

1. **Impact of Crack Location and Inclination:** Static deflection decreases as the crack moves away from the fixed end, with a deflection peak at an angle of 90°, highlighting significant influences of location and angle on beam dynamics.
2. **Influence of Relative Crack Depth:** A direct cor- relation exists between increased relative crack depth and static deflection due to reduced beam stiffness, emphasizing the need for monitoring crack progression for structural integrity.
3. **Uniformity in Frequency Responses:** Consistent de- crease in frequency with increasing relative crack depth across both Timoshenko and Euler-Bernoulli beam models underlines the reliability of these theories in structural dynamics.
4. **Differential Frequency Analysis:** Both models show frequency reductions with deeper cracks; however, response nuances depend on the crack inclination and proximity to the fixed ends under dynamic conditions.
5. **Effectiveness of Data Visualization:** Utilization of interaction and scatter plots effectively demonstrates how structural variables influence frequency, aiding in detailed behavioural analysis.
6. **Model Comparisons and Recommendations:**
   a. **Consistency and Applicability:** Similar frequency responses to crack depth and inclination suggest the applicability of both Timoshenko and Euler- Bernoulli models for predicting vibrational behaviours.
   b. **Dynamic vs. Static Conditions:** Timoshenko model is preferable for dynamic scenarios due to its inclusion of shear deformation and rotary inertia effects, while the Euler-Bernoulli model is suited for static conditions in civil engineering applications.
   c. **Theoretical and Practical Implications:** Findings support the robust theoretical foundations of beam theories, confirming their essential role in modern engineering design and analysis.

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