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Study of Walrasian Model Using Quantum Calculus

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Abstract
In this work, for the first time, we present the Walrasian Model in view of q-calculus as a novel perspective of economic dynamics. Though q-methods have been widely applied in mathematical physics, their use in microeconomic theory remains unexplored. In order to study this model, at first, we begin by outlining the foundational concepts of q-calculus, and then reformulate the Walrasian framework within this q-calculus set up. This approach is not only applied to solve the one-dimensional dynamic equation that illustrates the effectiveness through a classic example of microeconomics but also the numerical validation against classical datasets confirms the robustness of this method. Most importantly, we show, if the deformation parameter q tends to 1, the results converge seamlessly to those obtained via the classical calculus that bridges the new formulation with established theory. This study also highlights the potential of q-calculus as a powerful tool for advance research in economic dynamics and opens new directions for theoretical as well as applied economics.

1. Introduction

Quantum calculus, well known as q-calculus, is a rapidly emerging mathematical tool across the wide landscape of modern research [1-10]. In the last few decades, q-calculus approach is used in astrophysics, quantum mechanics, and statistical mechanics by offering a powerful and elegant alternative to traditional limit-based approaches [9-11]. Leibniz's classical calculus, the long-standing formulation of mathematical science, relies fundamentally on the concept of limits, but q-calculus operates without it. This key distinction unlocks a more direct and often more intuitive pathway to model discrete and complex systems [4-7]. In this approach, the parameter q acts not as an abstract mathematical symbol, but as a powerful fitment parameter, providing a unique lens to model systems with inherent and standard differentiability [7-10]. Thus, q-model offers a fresh, flexible formalism for tackling problems where traditional continuous models may fall short [2-6]. In this pioneering work, we propose a new ground by deploying this unconventional yet potent mathematical framework to re-examine a classical model of economic theory i.e. the ‘Walrasian Price Stability Model’. More specifically, the ‘Walrasian Price Stability Model’ is a classical model in microeconomic theory that describes a dynamic framework for analyzing the adjustment of market prices over time by considering the supply and demand imbalances. In addition, this model shifts away from static representations of market equilibrium by introducing a tâtonnement (continuous-time adjustment process) to determine the time evolution of prices toward equilibrium through decentralized interactions among market participants [14]. This model also emphasizes the gradual and iterative nature of price formation by relying on the idealized notion of a perfectly informed auctioneer who instantly identifies equilibrium prices [12-14]. It also uses differential equations to describe the rate and direction of price changes which is driven by the presence of excess demand or excess supply [12,14]. Thus, this mathematical approach gives a detailed exploration of the conditions under which markets stabilize or destabilize as well as partial insights into the feedback mechanisms and responsiveness resulted in the price adjustment process [14]. Though this model is abstract in formulation, the 'Walrasian Price Stability Model' has significant relevance for both theoretical and applied economics. Firstly, it underpins the study of general equilibrium that supports the development of computational economic models. Secondly, it also serves as a reference point for evaluating the adaptability and robustness of market systems. More conveniently, this model can indicate toward equilibrium rather than assuming its immediate realization. However, in this paper, we move beyond classical differential equations to model the dynamic process of price adjustment using the tools of q-calculus. This novel approach allows us to incorporate discrete, step-by-step adjustment mechanisms and explore economic stability in a new light. It also reveals the potential insights that remain hidden within the traditional continuous limit. This paper also represents the first application of q-calculus to this fundamental economic problem, opening a new avenue for multidisciplinary research with advanced mathematics and economic theory. Moreover, the q-framework also presents how economic coordination unfolds over time and may provide a more realistic understanding which can be explored further by deploying feasible dataset.

1.1. Fundamentals of q-Calculus

In this subsection, we are displaying a few basic definitions (as described in [1-5]) of q-calculus which are essential for this modelling. The q-number

$$[m]_q = \frac{q^m - 1}{q - 1} \qquad (1)$$

for any positive integer m and $\lim_{q\to 1}[m]_q = m$ Hence, $[0]_q = 0$ and $[1]_q = 1$. The quantum factorial (q-factorial) $[m]_q!$ is defined as,

$$[m]_q! = \prod_{m=1}^{m}[m]_q \text{, and} [0]_q! = 1. \qquad (2)$$

If $\mathcal{F}(t)$ is an arbitrary function, then the q-differential can be written as,

$$d_q\mathcal{F}(t) = \mathcal{F}(qt) - \mathcal{F}(t) \qquad (3)$$

and the q- derivative will be

$$D_q\mathcal{F}(t) = \frac{d_q\mathcal{F}(t)}{d_q t} = \frac{\mathcal{F}(qt) - \mathcal{F}(t)}{(q-1)t}. \qquad (4)$$

Note that,

$$\lim_{q\to 1} D_q\mathcal{F}(t) = \frac{d\mathcal{F}(t)}{dt}. \qquad (5)$$

Thus,

$$D_q t^m = \frac{(qt)^m - (t)^m}{(q-1)t} = [m]_q t^{\,m-1}. \qquad (6)$$

The Jackson q-integral (definite) is given by

$$\int_0^a \mathcal{F}(t)d_q t = (1-q)a\sum_{m=o}^{\infty} q^m\mathcal{F}(q^m a). \qquad (7)$$

Hence,

$$\int_0^a D_q\mathcal{F}(t)d_q t = \mathcal{F}(a) - \mathcal{F}(0). \qquad (8)$$

The small q-exponential function of "t" is (represented as $exp_q(t)$ is basically

$$exp_q(\mathcal{R}t) = \sum_{m=0}^{\infty}\frac{(\mathcal{R}t)^m}{[m]_q!}. \qquad (9)$$

And,

$$D_q exp_q(\mathcal{R}t) = \mathcal{R}exp_q(\mathcal{R}t). \qquad (10)$$

Whereas,

$$\int exp_q(\mathcal{R}t)d_q t = \frac{1}{\mathcal{R}}exp_q(\mathcal{R}t) + c\,. \qquad (11)$$

where $c, \mathcal{R}$ are real constants. This exponential does not have any inverse and therefore no logarithm and

$$exp_q(t_1)exp_q(t_2) = exp_q[(t_1 + t_2)],$$

if and only if $t_1 t_2 = q t_2 t_1$. An important rule for q-antiderivative is,

$$\int \frac{d_q\mathcal{F}}{\mathcal{F}} = \frac{q-1}{log(q)} log\mathcal{F} + d\,. \qquad (12)$$

Where d is constant of integration and in the limit $q \to 1$, all the definitions reduce to its classical form unambiguously.

2. Classical Walrasian Model

Classical Walrasian price stability model is a basic example of a dynamics model from the microeconomic theory. We suppose, the demand and the supply function of a commodity in a market are given by

$$\mathfrak{D}(t) = \alpha - \beta\mathcal{P}(t), \qquad (13)$$

$$\mathcal{S}(t) = \delta + \varepsilon\mathcal{P}(t). \qquad (14)$$

Where $\mathfrak{D}(t)$, $\mathcal{S}(t)$, $\mathcal{P}(t)$ represent the quantity demanded, quantity supplied and the price at time t respectively. $\alpha, \beta, \delta,$ and are ε real-valued parameters. Note that, the stability of the price-quantity equilibrium in the Walrasian price stability model is completely determined by the time path of the price variable. Generally, differential equation associated with this model can be represented by,

$$\frac{d\mathcal{P}(t)}{dt} = \dot{\mathcal{P}}(t) = \mathcal{k}[\mathfrak{D}(t) - \mathcal{S}(t)]. \qquad (15)$$

In (15), $\mathcal{k} > 0$ is an arbitrary constant of variation. Using (13) & (14), (15) can be written as,

$$\dot{\mathcal{P}}(t) = \mathcal{k}[\{\alpha - \beta\mathcal{P}(t)\} - \{\delta + \varepsilon\mathcal{P}(t)\}]. \qquad (16)$$

Rearranging, we can write,

$$\dot{\mathcal{P}}(t) + \mathcal{k}(\beta + \varepsilon)\mathcal{P}(t) = \mathcal{k}(\alpha - \delta). \qquad (17)$$

Thus,

$$\dot{\mathcal{P}}(t) + \mathcal{k}(\beta + \varepsilon)\left[\mathcal{P}(t) - \left(\frac{\alpha - \delta}{\beta + \varepsilon}\right)\right] = 0. \qquad (18)$$

Letting,

$$\mathcal{P}(t) - \left(\frac{\alpha - \delta}{\beta + \varepsilon}\right) = \mathcal{P}(t) - \Pi = \mathcal{P}'(t), \qquad (19)$$

And

$$\mathcal{k}(\beta + \varepsilon) = \mathcal{k}' \qquad (20)$$

(18) can be rewritten as,

$$\frac{d\mathcal{P}'(t)}{dt} + \mathcal{k}'\mathcal{P}'(t) = 0 \qquad (21)$$

Solving (21), we obtain,

$$\int_{\mathcal{P}'(0)}^{\mathcal{P}'(t)} \frac{d\mathcal{P}'}{\mathcal{P}'} = -\int_0^t \mathcal{k}'d\tau\,. \qquad (22)$$

Where, at $t = 0$, $\mathcal{P}'(0) = \mathcal{P}(0) - \Pi$ and thus using (19) & (20), we get,

$$\mathcal{P}(t) - \Pi = [\mathcal{P}(0) - \Pi]exp(-\mathcal{k}'t) \qquad (23)$$

Hence, finally,

$$\mathcal{P}(t) = \Pi + [\mathcal{P}(0) - \Pi]exp(-\mathcal{k}'t) \qquad (24)$$

Therefore, (24) depicts the relation with time and price variable in the Walrasian price stability model. Table 1 *shows* $\mathcal{P}(t)$ for different t with $\mathcal{P}(0) = 4$$

Table 1 $\boldsymbol{\mathcal{P}(t)}$ **for different t with** $\boldsymbol{\mathcal{P}(0) = 4}$$

(t)	$\mathcal{P}(t)$	Value of $\mathcal{P}(t)$
0.0	17-13	4.000
0.2	$17 - 13exp(-0.4)$	8.286
0.4	$17 - 13exp(-0.8)$	11.159
0.6	$17 - 13exp(-1.2)$	13.084
0.8	$17 - 13exp(-1.6)$	14.375
1.0	$17 - 13exp(-2.0)$	15.241
1.5	$17 - 13exp(-3.0)$	16.353
2.0	$17 - 13exp(-4.0)$	16.762
3.0	$17 - 13exp(-6.0)$	16.978
4.0	$17 - 13exp(-8.0)$	16.996
5.0	$17 - 13exp(-10.0)$	16.999

As, $\mathcal{k} > 0, \mathcal{P}(t) = \Pi$ for $t \to \infty$ and Π is termed as equilibrium price. So, in this model, the price-quantity equilibrium is dynamically stable. For

Table 2 Detailed values of $\mathcal{P}(t)$ for different t

(t)	$\mathcal{P}(t)$ for $\mathcal{P}(0)$=5$	$\mathcal{P}(t)$ for $\mathcal{P}(0)$=6$
0.0	5.000	6.000
0.5	12.585	12.953
1.0	15.376	15.511
1.5	16.403	16.452
2.0	16.780	16.799
2.5	16.919	16.926
3.0	16.970	16.973
3.5	16.989	16.990
4.0	16.996	16.996
4.5	16.999	16.999
5.0	16.999	17.000

example, let us suppose, $\mathcal{P}(0) = 4\,\$$, $\alpha = 95, \beta = 2,\ \delta = 10,\ \varepsilon = 3,\ and\ \ \mathcal{k} = \ 0.4.$ For equilibrium price, $\mathfrak{D}(t) = \mathcal{S}(t)$, which gives, $\Pi = 17$, and the equilibrium quantity $\mathfrak{D}_e(t) = \mathcal{S}_e(t) = 61$. Then, for these numerical inputs (24) can take the form as,

$$\mathcal{P}(t) = 17 + [4 - 17]exp(-2t) \qquad (25)$$

3. Walrasian Model in View of Quantum Calculus

In view of "q" -calculus, (21) can be written as,

$$\frac{d_q\mathcal{P}'}{d_q t} + \mathcal{k}'\mathcal{P}'(t) = 0 \qquad (26)$$

Hence,

$$\int_{\mathcal{P}'(0)}^{\mathcal{P}'(t)} \frac{d_q\mathcal{P}'}{\mathcal{P}'} = -\int_0^t \mathcal{k}' d_q\tau\,. \qquad (27)$$

Setting $\mathcal{k}_q = \mathcal{k}'\left(\frac{log(q)}{q-1}\right)$, using (12) & (23), and remembering the definition of $[1]_q$, we obtain,

$$\mathcal{P}_q(t) - \Pi = [\mathcal{P}(0) - \Pi]exp(-\mathcal{k}_q t) \qquad (28)$$

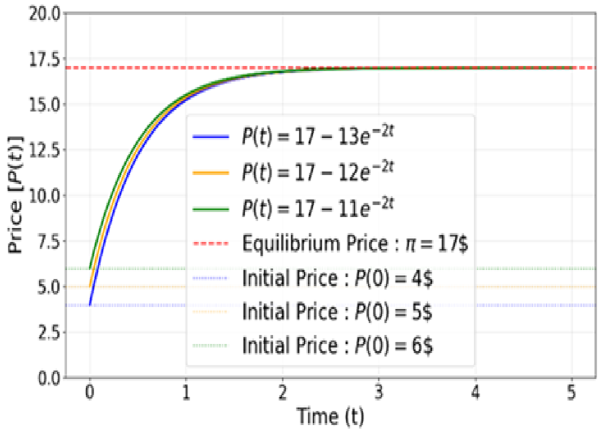

Figure 1 Walrasian Price Stability Model: Dynamic Adjustment to Equilibrium for Different Initial Prices

It is clear from (28), for $q \to 1$, $\mathcal{P}_q(t) \to \mathcal{P}$ and it changes the form to its original classical form. Additionally, using (28) we can obtain different values of $\mathcal{P}_q(t)$ for different value of t and q whenever all the inputs are same. This indicates that q may serve as a fitment factor for real data set. Figure 2 shows PYTHON plot that depicts $\mathcal{P}_q(t)$ t for different values of q and the price is forecasted either underestimated or overestimated for any instant t.

4. Discussion

In figure 1, the classical P(t) is plotted with t by taking initial price P(0)= 4 $,5$, 6$ and $,α=95,β=2,δ=10,ε=3," and" k= 0.4. For equilibrium price, D(t)= S(t), which gives, Π=17, and the equilibrium quantity D_e (t)= S_e (t)=61. It is also important to note that in all the figures P(t) is written as P(t) for convenience. It is clear that after a certain time interval, P(t) reaches the equilibrium depending upon the supply and demand. Though, the practical situation is not so simple but, this model gives an outline about the time evolution of price. On the other hand, in figure 2, we plot time evolved price P_q (t) with t. In this plot we consider $\mathcal{k}_q = \mathcal{k}'\left(\frac{log(q)}{q-1}\right)$ for plotting the exponential function as mentioned in (28). It is observed that the nature of curves is same as of the classical average plot but the q-solution gives different values of P_q (t) for different values of t. Thus, practically, this predicted for any particular q value may be fitted for a particular price, and supply-demand set up that can be explored further. Note that, in both the cases the saturated prices are same which indicates the validation of q-solution with the classical one. At any particular instant, P_q (t) for different values of q are depicted in figure 3. It shows that P_q (t) can have larger or smaller predicted values than the classical Figure 2 Walrasian Price Stability Model with q-Deformation values depending on the values of q as shown in figure 3.

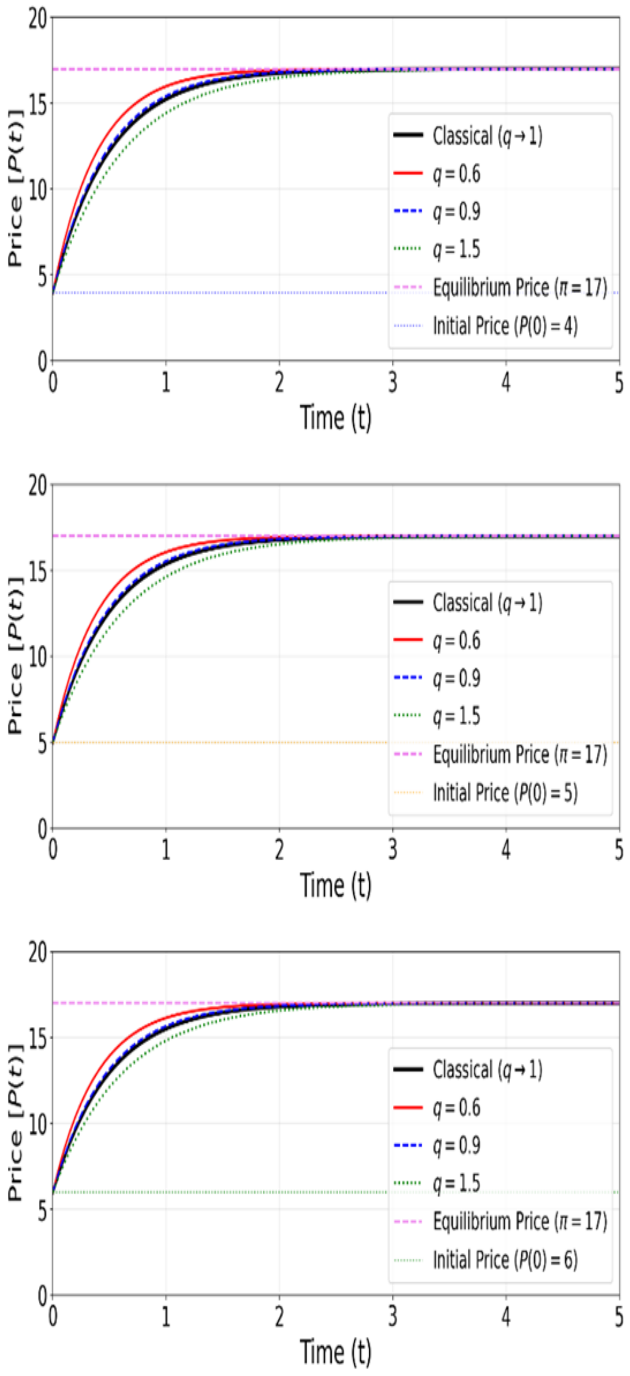

Figure 2 Walrasian Price Stability Model with q-Deformation

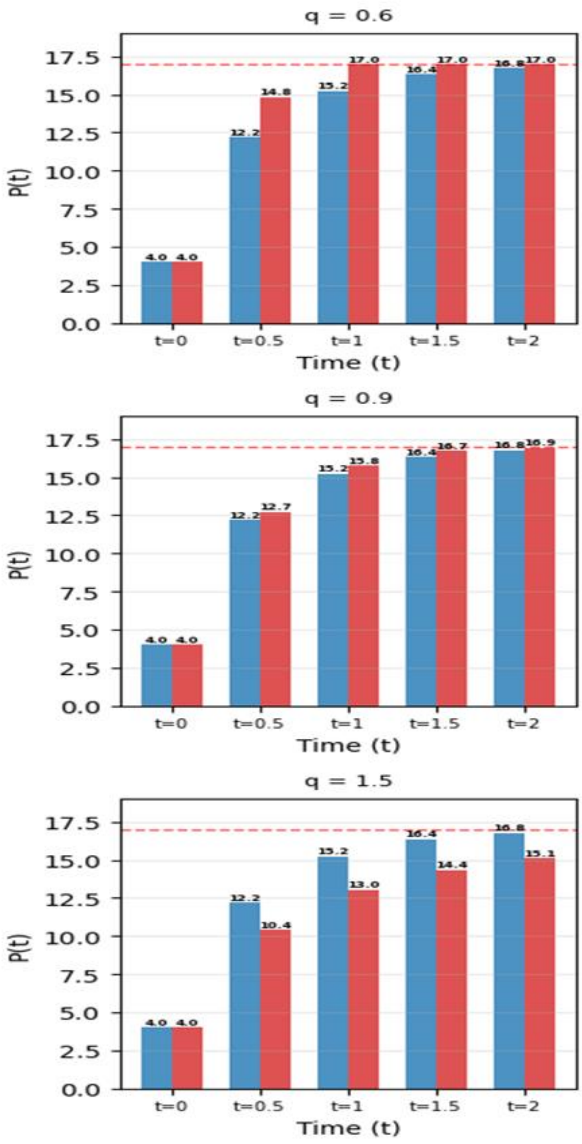

Figure 3 The bar chart shows standard (Blue bar) and deformed (Red bar) P(t) for different values of t

Conclusion

Here, we study the Walrasian model under the framework of q-calculus that enriches the analytical scope of this model by creating a bridge between purely discrete and purely continuous dynamics. In this q-framework the q-derivative has served as a tool to represent scale-dependent price adjustments, where changes are neither completely smooth nor entirely stepwise. Additionally, the use of the q-analogs for stability and equilibrium, offers a refined understanding of how markets return to balance after disturbances. More clearly, according to this model, price may vary depending upon several parameters but it can be predicted by setting the value of q. Though, it is a matter of further detailed exploration, but for a particular market participants (demand-supply) scenario, we may search for the value of q for predicting the price at a particular time instant. In this way, the q-calculus approach extends the Walrasian model into a more versatile framework, better suited for analyzing the complexities of modern economic systems. It is also worth noting that as $q \to 1$, (28) reduces to (24) which confirms the validity of this q-model with the corresponding classical one. However, it is also anticipated that this mathematical framework can add a good pedagogical example in mathematics as well as mathematical economics.

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