Multilevel Space-Time Codes on Hoyt Fading Channel

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Abstract

Multilevel space-time trellis codes with multiple inputs and multiple outputs under the presence of the Hoyt fading in the communication channel are illustrated. The spectral efficiency shoots higher substantially if a code is spread out at more than one level, due to the fact that bandwidth used to transmit more symbols is same as that used to transmit less symbols in case of a single level system. Complexity of decoding a system with more than one level increases but using a multistage decoder it can be dealt with. The resultant system with multiple levels of coding and multiple stages of decoding maintains the balance between spectral efficiency and decoding complexity with desirable error performance.

Keywords: Space-time Trellis Codes, Multilevel Codes, Hoyt Fading, Multistage Decoding

1. Introduction

Wireless communication was the long last missing piece of the puzzle in communicating over longer distances [1]. Since its advent, the main barrier is not the wireless transmission and reception, but the quality and quantity of the data being transmitted, in short, a good wireless communication system with best error performance and highest spectral efficiency is required which can be made but at cost of more bandwidth usage [2]. Everyone needs more and more bandwidth every single minute. But the bandwidth is nearly constant and can’t be increased without incurring additional costs. So, to achieve better performance repeatedly with same bandwidth, systems with multi-input multi-output can be used. In this paper, a multi-level multi-input multi-output (MIMO) system is represented [3]. The base code used are the space-time trellis codes (STTCs). STTC can simultaneously achieve good coding gain, spectral efficiency and diversity improvement at the loss of exponential rise in decoding complexity [4]. Multilevel coded (MLC) modulation, is used in conjunction with the STTCs at every level to decrease the complexity [5]. The combined form of STTC and MLC are known as multilevel space-time trellis codes (MLSTTCs) [6]. MLSTTCs are capable of simultaneously providing higher throughputs, coding gain, bandwidth efficiency, and diversity improvement with significantly reduced decoding complexity, especially for larger constellations. The error performance of the overall system is evaluated over Hoyt fading channel. Hoyt fading is statistical fading model based on the Hoyt distribution. In Nakagami-\(m\) distribution, if the parameter \(m\) is confined to the unit interval such that for another parameter \(q = m, 0 < q < 1\), we get a Nakagami-\(q\) distribution, also known as Hoyt distribution [7].

2. System Model

We consider a multi-input multi-output wireless system as shown in Figure 1, with \(n_T\) transmit antennas and \(n_R\) receive antennas. The symbol transmitted at time \(t\) by the \(i^{th}\) transmit antenna is
denoted by $Q^i_t$, $1 \leq i \leq n_T$. The channel exhibits Hoyt fading over the frame duration, means it is constant over one frame and varies independently between frames. Perfect Channel State Information (CSI) is available at the receiver only [8,9].

The received signal at time $t$, at the $j^{th}$ receive antenna is a noisy superposition of independently faded versions of the $n_T$ transmitted signals and is denoted by $r^j_t$, $1 \leq j \leq n_R$. The complex baseband output of the $j^{th}$ receive antenna at time $t$ is given by,

$$r^j_t = \sum_{i=1}^{n_T} h^j_{ij} Q^i_t + \eta^j_t \tag{1}$$

where, $h^j_{ij}$ is the path gain between the $i^{th}$ transmit and $j^{th}$ receive antennas and $\eta^j_t$ is the noise associated with the $j^{th}$ receive antenna at time $t$. The path gains, $h^j_{ij}$, are modeled as samples of independent non-complex Hoyt random variables with factor $q = 0.5$. The noise quantities are samples of independent complex Gaussian random variables with zero mean and variance of $N_0/2$ per dimension.

In matrix form, (1) can be represented as,

$$\begin{bmatrix} r^1_t \\ r^2_t \\ \vdots \\ r^n_R t \end{bmatrix} = \begin{bmatrix} h^1_{11} & h^1_{12} & \cdots & h^1_{1n_R} \\ h^2_{11} & h^2_{12} & \cdots & h^2_{1n_R} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_R1} & h_{n_R2} & \cdots & h_{n_Rn_R} \end{bmatrix} \begin{bmatrix} Q^1_t \\ Q^2_t \\ \vdots \\ Q^n_R t \end{bmatrix} + \begin{bmatrix} \eta^1_t \\ \eta^2_t \\ \vdots \\ \eta^n_R t \end{bmatrix} \tag{2}$$

In compact form, (2) can be represented as,

$$r_t = H_t Q_t + \eta_t \tag{3}$$

where, \( r_t = (r^1_t, r^2_t, \ldots, r^n_R t)^T \), \( Q_t = (Q^1_t, Q^2_t, \ldots, Q^n_R t)^T \), \( \eta_t = (\eta^1_t, \eta^2_t, \ldots, \eta^n_R t)^T \) and \( H_t \) is the $n_R \times n_T$ channel matrix whose $j^{th}, i^{th}$ entry is represented by $h^j_{ij}$.

### 2.1 MLSTTC Encoder

Similar to a multi-resolution modulation (MRM), here the encoder divides the constellation into equal divisions with parent-child like tree type graph association whereby each division has further sub-divisions. The bits at the input of the encoder are encoded and then mapped to MRM constellation with 2$m$ points. The most significant encoded input bit is mapped to the main division or cluster. The least significant encoded input bit is mapped to the sub-divisions or sub-clusters within main division or main cluster and so on. This gives $L$ resolutions for an $M$-QAM constellation, whereby, $M = 4L$, with each resolution as a 4-QAM constellation, reducing to $L$ component codes being used to encode the input data. System model of a MLSTTC system is shown in Figure 1, with component codes denoted as $C(1), C(2), \ldots, C(L)$. STTCs are employed as main the component codes. The output of $C(L)$ is denoted as $x_t(L)$. Partition criteria is based on the MRM is used on each of the $n_T$ transmit antennas dividing the signal constellation into tree-like divisions, with each division having its own sub-divisions. Each point in the underlying $M$-QAM constellation, at time $t$, the symbol transmitted from the $i^{th}$ transmit antenna is given by,

$$Q^i_t = d_{x_t(1)} x(1)^i + d_{x_t(2)} x(2)^i + \cdots + d_{x_t(L)} x(L)^i \tag{4}$$

where, $d_{x_t(1)}, d_{x_t(2)}, \ldots, d_{x_t(L)}$, are the cluster distances corresponding, $x_t(l)^i$ a 4-QAM symbol generated by the $l^{th}$ component code, $1 \leq l \leq L$.

### 2.2 MLSTTC Decoder

Multi-stage decoder with $L$ stages is shown in Figure 1. It is used to decrypt and/or decode the data received from the Hoyt faded channel [8]. Firstly, it decodes the output of $C(1)$. The approximated value of $x_t(1)$ denoted by $\hat{x}_t(1)$ is sent to the next stage to decode the value of $x_t(L - 1)$ to get $\hat{x}_t(L - 1)$ and so on [9]. The final decoding stage uses the estimates obtained from levels 1 to $L - 1$, viz. $\hat{x}_t(1), \hat{x}_t(2), \ldots, \hat{x}_t(L - 1)$ to get $\hat{x}_t(L)$.
The received signal at the \( j \)th receive antenna at time \( t \) is given by,

\[
r_t^j = \sum_{l=1}^{L} \sum_{i=1}^{n_T} h_{j,i}^l d_x(l)x(l)^i + \eta_t^j \tag{5}
\]

For an \( L \)-level MLSTTC system, if the decoding starts from stage \( L \), the branch metric at stage \( k \), where \( 1 < k \leq L \), for a transition labeled \( x_t(k)^1, x_t(k)^2, \ldots, x_t(k)^i \) can be calculated as

\[
\max \left\{ x_{d(l)}, \left| \sum_{j=1}^{n_R} r_t^j - \sum_{l=1}^{k} \sum_{i=1}^{n_T} h_{j,i}^l d_x(l)x(l)^i \right| \right. \\
- \sum_{p=k+1}^{L} \sum_{i=1}^{n_T} h_{j,i}^l d_x(p)\hat{x}(p)^i \right\} \tag{6}
\]

In the final stage, the Viterbi algorithm is used to decode the path with the lowest accumulated metric.

3. System Performance

System is evaluated for error performance with 2 transmit and 1 receive antenna, 2 transmit and 2 receive antennas, 2 transmit and 4 receive antennas. Bi-level coding \((L = 2)\) and a 16-QAM constellation is partitioned two times using 4-way partitions into subset of constellation points. The subset distances \( d_{x(1)} = 2 \) and \( d_{x(2)} = 1 \). Component code \( C(1) \) & \( C(2) \) are 4-QAM STTCs designed for 2 transmit antennas using trace criterion with trellis diagram shown in Figure 2 [9]. Hoyt fading channel model is assumed over the frame duration, which is constant over a frame and varies independently between frames. A total of 130 input symbols per frame are considered with 1000 frames in total. The number of input bits per symbol is 2, thus, the spectral efficiency of 4 bits/sec/Hz is achieved. Perfect channel state information is assumed to be present at the receiver only [8]. The effect of receive diversity using 2 transmit antennas and various counts of receive antennas is evaluated on the error performance. Frame error rate (FER), symbol error rate (SER) and bit error rate (BER) of the code are calculated. The error performance results are drawn against the signal-to-noise ratio (SNR) per receive antenna. The FER, SER and BER performance of the MLSTTC system described here in this section is shown in Figure 3, 4 and 5 respectively. The x-axis in Figure 3, 4 and 5 denotes the SNR from 0 dB to 40 dB. The y-axis in Figure 3, 4 and 5 corresponds to the respective error rate.
At 14 dB SNR, the SER is 0.19708, 0.017292 and 0.00021538 for 1, 2 and 4 receive antenna respectively. Similarly, in case of symbol error rate, again the same result is deduced. The results of symbol error rate depict the increase in performance of the system with increase in the number of receive antennas.

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At 14 dB SNR, the BER is 0.12075, 0.010488 and 0.00012308 for 1, 2 and 4 receive antenna respectively. The results show that increasing the number of receive antennas yield a significant performance gain in terms of bit error rate.

**Conclusions**

In this paper, the MLSTTCs were used with Hoyt fading channel. MLSTTC give efficiency in terms of bandwidth requirements concurrently with improvement in diversity and coding gain and that too with reduced decoding complexity, even for larger constellations. Simulation results were provided for error performance of MLSTTCs with 2 transmit and different number of receive antennas over Hoyt fading channel. The results are in accordance with the idea of receive diversity yielding a significant performance gain.

**References**