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Rough topological space using equivalence relation

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Abstract
To generalise the basic rough set definitions, the topology induced by equivalence relations is used. The proposed topological structure opens the way for the implementation of a broad range of topological facts and techniques in the granular computing process, including the introduction of the definition of topological membership functions that incorporates the concept of rough and fuzzy sets. There is an overlap between rough set theory and several other theories dealing with incomplete knowledge
Keywords: Sets, Rough Sets, Topological Space and Free Sets

1. Introduction
It is possible to consider rough set theory as a new mathematical method for imperfect data analysis. In many fields, such as decision support, engineering, the environment, finance, medicine and others, the theory has found applications. The theory provides an attempt to deal with confusion or vagueness. The Rough set theory has drawn the attention of many scientists and practitioners who have ultimately contributed to its development and applications.

The Preliminaries:
This section will briefly review basic concepts and outcomes of the rough sets dependent relationship and rough topological space with equivalence relationship and some important definitions.[1-5]

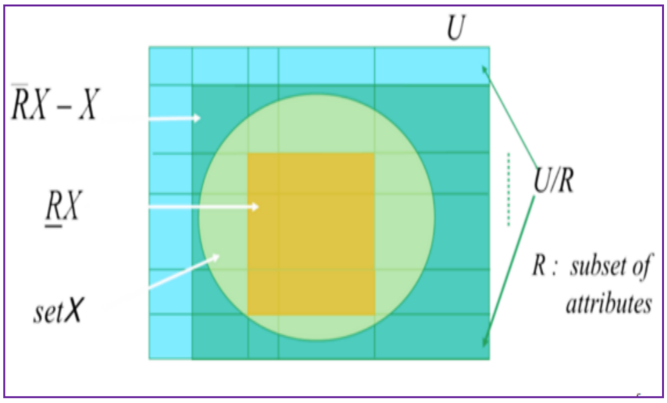
1.1 Definition
Let X is subset of U, let R be a relation of equivalence to U. Define the following, then,the Lower Approximation of X with regard to R is the set of all objects that can be identified with certainty as members of X with regard to R. It is defined by

$$\underline{R}\,(x) = \{\ x\ :\ R[x]\ \subseteq\ X\ \}$$

1.2 Definition
The upper approximation of X with respect to R is the set of all objects that can be identified with certainty with respect to R as potential members of X. It is defined by

$$\overline{R}(X) = \{x : R[x] \cap X \neq \emptyset\}$$

Lower and upper approximation diagram



1.3 Definition
The difference between the upper and lower approximations is the boundary region of the set.

Intuitively, the boundary region of the set consists of all elements that, by the use of available information, cannot be identified uniquely as a set or its complement. It is defined by

$$BN_B(X) = R^*X – R_*X$$

A set, if it has a non-empty boundary area, is said to be a rough set. If the boundary region is empty, the Crisp or Exact Set set is the set.

1.4 Definition:

A topological space [3] is a pair (X, T) consisting of a collection of X and a family of subsets of X that satisfy the following requirements:

i) ∅, X ∈ T
ii) Under an arbitrary union, T is closed.
(iii) Under the finite intersection, T is closed.

Examples:

1. Consider the following set consisting of three points,

X = {a, b, c} and determine if the set T = {∅, X, {a}, {b}} satisfies the requirements for a topology. This is, in fact, not a topology because the union of the two sets and {a} and {b} is the set {a, b}, which is not in the set T.

2. See the all possible topologies on X = {a, b}.
   1. T = {∅, {a} {b}}
   2. T = {∅, {a}, {a, b}}
   3. T = {∅, {b}, {a, b}}
   4. T = {∅, {a}, {b}, {a, b}}

1.5 Definition

Let A = ($A_{L,}$ A_U)be any rough subset of a RTS(X ,T)Where X=(X_L ,X_U) and
T = (T_L , T_U) .

Then A is said to be lower rough open if the lower approximation of A is in the lower rough topology that is $A_{L,} \in T_L$ .Also A is said to be upper rough open if the upper approximation of A is in the upper rough topology that is $A_U \in T_U$. A is said to be rough open iff A is lower rough open and upper rough open. That is A=($A_{L,}$ A_U) is rough open with respect to the RTS (X ,T) iff $A_L \in T_L$ and $A_U \in T_U$

1.6 Definition:

A reference, R on a non-empty set S is said to be a relation of equivalence on S if

(a) Reflexiveness (xRx for any x object)
(b) Symmetry (If yRx ,xRy then)
&
(c) Transitivity (where xRy and yRz then are xRz)

For the equivalence R with the symbols R[X] and R[x] = {y ∈ S: y Rx}, we denote the equivalence class of an element x ∈ S with respect to the equivalence R

Example:

Conjugacy is a relationship of equivalence in G.

1.7 Definition:

If x, y G then y is said to be a conjugate of x in G if an element c G exists such that $y = c^{-1} x c$

Example:

- X ~ Y if and only if every rational number is x-y

Observations:

- A relation of equivalence causes a partitioning of the universe.
- The partitions can be used to create the universe 's new subsets.
- The same value of the decision attribute is assigned to subsets that are most often of interest.

Example:

- All subsets of Z integers are declared open, so Z in the so-called discrete topology is a topological space.

1.8 Definition:

Topological equivalence here, Topologists study (topological) spaces up to homeomorphism, just as algebraists study groups up to isomorphism or matrices up to linear conjugacy.[6-8]

Topological space Fundamental Study:

A map f: X ¬ Y between topological spaces is a homeomorphism if it is continuous and bijective with the inverse continuous. If there is an X-Y homeomorphism, then it is said that X and Y are homeomorphic or often topologically identical. A property of a topological space that is the same is said to be a topological invariant for any two homeomorphic spaces. Obviously, the relationship of being homeomorphic is a relationship of equivalence (in the technical sense: reflexive, symmetric, and transitive). Topological spaces are thus divided into classes of equivalence, often referred to as homeomorphic classes. In this relation, the topologist is often represented as a person who (since these two objects are homeomorphic) can not distinguish a coffee cup from a doughnut. In other words, from the intrinsic point of view, two homeomorphic topological spaces are identical or indistinguishable in the same way as isomorphic groups are indistinguishable from the abstract group theory point of view, or two conjugate n x n matrices are indistinguishable as linear n-dimensional vector space transformations without a fixed vector space.

1.9 Definition:

Relation System X= (U,R) Where U is universal set and R is equivalence relation over which Partition for U family of sets $X_1, X_2, \ldots X_n$ such that $X_i \subseteq U$ then $X_i \cap X_j = \varphi$ for $i \neq j$

Example:

U ={1,2,3,4,5,6,7,8,9} Then U is dived in to two classes even and odd then U/R=

[{ 1,3,5,7,9},{2,4,6,8,}]

Here X_1= [{ 1,3,5,7,9}X_2 = [{2,4,6,8,}]

Such that $X_1 \cap X_2 = \varphi$, $X_1 \cup X_2 \neq \varphi$

Properties :

1.$R_* \subseteq X \subseteq R^*$

2.$R_* \varphi = R^* \varphi = \varphi$

3.$R_* U = R^* U = U$

4. $R_* (X \cup Y) = R_* X \cup R^* Y$

5. $R_* (X \cap Y) = R_* X \cap R^* Y$

6. $X \subseteq Y$ then$R_* X \subseteq R_* Y$

7.$X \subseteq Y$ then $R^* X \subseteq R^* Y$

8.$R_*(-X) = -R_* X$

9$R^* (-X) = R^* X$

2. Definition:

Let us consider $A= (A_L, A_U)$,$B= (B_L, BU)$, $C= (C_L, C_U)$ $D= (D_L, D_U)$, $K= (K_L, K_U)$ any five arbitrary rough subsets of X. Les us consider the set

U= {A,B,C,D,K} are the vitamins.Then the set of vitamins classified into carrot (X_1),spinach(X_2), orange(X_3) , dairy products (X_4) , sweet potato(X_5), beans(X_6), eggs(X_7) , fish(X_8) , meat(X_9) sun (X_{10}) grains(x_{11})peas(X_{12}) lemon(X_{13})

Here A, B, C,D, K are called attributes

and X_1 , X_2, X_3 , X_4 , X_5, X_6 are called objects

Set A={X_1, X_2 ,X_3,X_4 , X_5}

Set B ={X_2, X_4 ,X_6,X_7 ,X_8,X_{11} ,X_{12}}

Set C ={X_3,X_{13}}

Set D = {X_{10}}

Here $A \cap B$ = { X_1,X_2,X_4}

$A \cap D$= { }

$A \cap K$ = { X_1,X_2,X_4}

$A \cup C \cup D$ = {X_1,X_2,X_3,X_4 , X_5,X_{10} ,X_{13}}

2.1 Theorem:

Every subset of a Discrete Rough Topological Space is rough open.

Proof: Consider any subset $A= (A_L,A_U)$of the Discrete Rough Topological Space(X,T) . Where $X= (X_L,X_U)$, $T= (T_L,T_U)$. Being the lower approximation of A, $A_L$is an exact subset of $X_L$and hence $A_L \in T_L$ therefore A is lower rough open. Also being the upper approximation of A, A_U is exact subset of X_U and hence $A_U \in T_U$ therefore A is upper rough open. That is $A= (A_L,A_U)$ is lower and upper rough open and therefore rough open subset of X. Since A is arbitrary, every subset of a Discrete Rough Topological Space is rough open.

Conclusion:

We found that every Rough topological space satisfies equivalence relation in this paper.

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