New Insights into Image Restoration Using Filter Analysis and Noise Models

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Abstract

Image restoration, by eliminating noise and blur from an image, restores the original image. In certain cases, image blur is inevitable, and to eliminate blur caused by camera shake or radar imaging or to remove the effect of image system reaction, etc. There are many suggested methods for noise removal and our paper will investigate and address various models of noise and blur and methods of restoration. There are numerous techniques developed, the most efficient being the Wiener filter and is the fundamental noise reduction approach. Wiener filters may cause some undesired effects in image restoration (significant degradation in quality). Various techniques and models are approached in the establishment of the power spectrum of noise and undegraded images. In terms of noise reduction and image restoration, this paper studies the Wiener filter's assumption and quantitative performance improvement. The SNR is improved considerably. But noise reduction is directly proportional to image degradation. To counter this, we must have prior knowledge of the original image by some PDF (Probability Distribution Function).

Keywords: Image Processing, Image Restoration, Noise Models, Probability Distribution Function, Power Spectrum, Filters

1. Introduction

Being able to communicate is one of the most important blessings in life. This communication is very important to make others understand whatever we want to convey in a way understandable to them. Communication can be done in any way but images are a very efficient way of conveying our message to others. Images play a very important role in our day-to-day life. They contribute a very huge proportion to the important data like the images sent by satellites, the different treatments being done in hospitals, and many more such applications. Any kind of loss in these images can be a very big loss in the data set. Each time when the data is transmitted in the form of images from a sender to receiver, noise is being added to the images unwantedly. These create problems not only at a small scale but also at larger scales. Thus, there arises the need to bring up some techniques to remove the noises added to images unwantedly. This has contributed to the development of different techniques for image processing. It is a way to perform some image operations, to obtain an enhanced image, and then we can extract some useful image information. One of the easiest and most enticing fields of digital image processing is image enhancement in the long run. The philosophy behind the different techniques of enhancement is to highlight certain characteristics of interest in an image and they can vary according to different users. Image restoration is also similar, but it explicitly deals with improving an image's appearance. Image
restoration techniques appear to be based on statistical or probabilistic image degradation models, unlike image enhancement techniques. Noise in an image is any kind of image signal degradation caused by any kind of external interference when an image is transmitted via satellite, wireless, and network cable from one location to another location. Although different solutions are available for de-noising an image, the scope of improvement remains: median filter, Gaussian filter, Kuan filter, morphological filter, homomorphic filter, bilateral filter, and wavelet filter \[1\]. The different aspects of image restoration will be discussed in this article.

2. Materials and Methods

Image restoration is the process of obtaining back an image from its own blurred or noisy version. It is very major in image processing. The quality of the restored image, the algorithm’s computational performance, and the estimation of required parameters such as the point-spread function are the key points to consider in image restoration (PSF). Unwanted data that may decrease the contrast that deteriorates the shape or size of the image objects and the blurring of the image edges may end up as noise. Noise occurs in images due to shortcomings of image acquisition devices and image developing mechanisms. They can also occur due to the environment or the physical nature of the system.

2.1. Types of Noise Models and Degradation Theory:

2.1.1. Degradation Model

Images are degraded by Degradation function \(H\) and a certain noise \(n(x, y)\). It can be modelled as: The original input is a two-dimensional input in the form the function \(f(x, y)\). The degradation function \(H(h(x, y))\) is being added to the input \(f(x, y)\). Apart from the degradation function some noise \(n(x, y)\) is also added. The resultant output is \(g(x, y)\) and is fed into the Restoration Filter.

2.1.2. Blur Model

2.1.2.1 Gaussian Blur:

It is a kind of blurring technique which uses a Gaussian function for changing each pixel.

\[
g(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3}
\]

Where \(x=\text{Distance from the origin and } \sigma=\text{Gaussian distribution’s standard deviation}\)

Thus, we obtain the output \((I(x, y))\) \[2\]

In Spatial Domain \(I(x, y) = h(x, y) * f(x, y) + n(x, y)\)

In Frequency Domain \(I(u, v) = H(u, v) \cdot f(u, v) + n(u, v)\)

here, \(g(u, v) = h(u, v) \cdot f(u, v)\)

2.1.2.2 Defocus Blur:

It is a type of blur in which the scene which is observed is not the focus. Its frequency response is:

\(H(u, v) = e^{-\frac{(u^2+v^2)}{2\sigma^2}}\)

2.1.2.3 Rectangular Blur:

In this form of blur, in a particular rectangular field, the object is blurred. Blur in the image can be defined on any component, it can be circular and rectangular based on this.

2.1.2.4 Motion Blur:

It happens when there is a relative movement between the camera and the pictured object.

2.1.3. Noise Model

There are various kinds of Noise Models that predict what kind of noise is possible. \[4\]

2.1 Restoration Techniques:

2.2.1 Mean Filter

It is a filter and can be applied very quickly to smooth the images, i.e. reducing the sum of difference in intensity between one pixel and the other. It is also used in image noise reduction. \[5\]
### Noise models

<table>
<thead>
<tr>
<th>Noise models</th>
<th>PMF (Probability mass function)</th>
<th>Mean and Standard deviation</th>
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<tr>
<td><strong>Gaussian</strong> (Thermal, Sensor Noise)</td>
<td>( p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} )</td>
<td>( \bar{z} = \text{Mean} )</td>
</tr>
</tbody>
</table>
| **Rayleigh** (Range Imaging)              | \( p(z) = \begin{cases} \frac{3}{2} (z-a)e^{-\frac{(z-a)^2}{b}} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases} \) | \( \bar{z} = a + \sqrt{\frac{\pi b}{4}} \)  
\( \sigma^2 = \frac{b(4-\pi)}{4} \) |
| **Exponential** (Laser Imaging)           | \( p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \) | \( \bar{z} = \frac{1}{a} \)  
\( \sigma^2 = \frac{1}{a^2} \) |
| **Salt and Pepper** (Faulty Components)   | \( p(z) = \begin{cases} Pa & \text{for } z = a \\ Pb & \text{for } z = b \\ 0, \text{otherwise} \end{cases} \) | If either Pa or Pb is equal to zero then assumes unipolar nature. |
| **Uniform**                               | \( p(z) = \begin{cases} ae^{-az} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases} \) | \( \bar{z} = \frac{a+b}{2} \)  
\( \sigma^2 = \frac{(b-a)^2}{12} \) |

![Probability density functions of the Gaussian, Laplacian, and Uniform distributions](image)

**Fig: 1.** Probability density functions of the Gaussian, Laplacian, and Uniform distributions

#### 2.2.2 Median Filter

A Median filter is a spatial filter sliding-window. In general, the filter is used to decrease noise from the image. As it retains the useful information of an image, it is better than the Mean Filter. This class of filters belongs to the class of smoothing filters that retain the edges and are non-linear filters. The soft and sharp details are kept intact by these filters. The drawbacks of such filters are that they break up image edges in the presence of small signal-to-noise ratios and thus create false noise edges and do not eliminate medium-tailed (Gaussian) noise distributions as well.\[^{[6]}\]

#### 2.2.3 Inverse Filter
The inverse filter is a deconvolution restoration technique, which means that it is highly feasible to recover the image by inverse filtering or generalization when the image is distorted by a known lowpass filter. We know that,

**In Frequency Domain,**

\[ I(u, v) = h(u, v) \cdot f(u, v) + n(u,v) \]

### 2.2.3.1 Image Restoration Using Generalized Inverse Filter

The quickest and easiest way to restore that is by inverse filtering. The benefit of the inverse filter is that in the absence of noise, it needs only the blur PSF as a priori information and that it allows for complete restoration. Since, noise is absent the above expression, the above expression becomes:

\[ I(u, v) = h(u, v) \cdot f(u, v) + n(u, v) = 0 \]

Now, for obtaining the undegraded image we have:

\[ f(u, v) = I(u, v) / h(u, v); f(u, v) = \text{undegraded image} \]

### 2.2.3.2 Some Disadvantages and facets of using inverse Filter:

1. The inverse filter does not exist since, at selected frequencies, \( h(u, v) \) is zero (u, v). That is the case for both the blur of linear motion and the blur of out-of-focus.

2. Second, even if the spectral representation \( h(u, v) \) of the blurring function does not actually go to zero but becomes weak, the second term known as the inverse filtered noise will become very large. As they are high-pass filters, inverse filtered images are dominated by highly amplified noise.

### 2.2.4 Wiener Filter

A variety of restoration filters, known as least-square filters, have been developed to solve the noise sensitivity problem of an inverse filter. Wiener filters consist of both the degradation function and statistical characteristics of noise into the process of restoration. The inverse filter is stronger. It interpolates into the restoration process both the deterioration and the noise statistical function. We treat image and noise as random procedures here.

The noise considered here is the AWGN (Additive White Gaussian Noise). Also, Wiener Filter requires a prior knowledge of the power spectrum of noise \( n(u, v) \) as well as the image \( f(u, v) \).
\[ f'(u, v) = \left[ S_r(u, v) \cdot h^*(u, v) / S_r(u, v) \cdot |h(u, v)|^2 + S_n(u, v) \right] \cdot I(u, v) \]

**Where,**

- \( h(u, v) = \text{Degradation Function}; \)
- \( h^*(u, v) = \text{Complex conjugate of } h(u, v); \)
- \( |h(u, v)|^2 = h^*(u, v) \cdot h(u, v) \)
- \( S_n(u, v) = |n(u, v)|^2 \quad S_r(u, v) = |f(u, v)|^2 \)
- \( f'(u, v) = \text{Estimated image} \)

![Image comparison](image.png)

**Fig. 3. Image comparison**

3. Results and Discussions

3.1 Implementation of Wiener Filter:

![Diagram](diagram.png)

**Fig. 4. Implementation of Wiener Filter:**

\[ f'(u,v) = [h(u,v) \cdot h^*(u,v)] / [h(u,v) \cdot |h(u,v)|^2 + k] \cdot I(u,v) \]

The ratio of \( S_n(u, v) / S_r(u, v) \) is called the noise to signal power ratio and is abbreviated with \( k \). If, for all the related values of \( u \) and \( v \), the noise power spectrum is zero, this ratio becomes zero and the Wiener filter is converted to the inverse filter. For better image reconstruction, the image which is approximated should be very close to the undegraded image. For obtaining the undegraded image, the value of \( k \) should be maximized to a great extent. The ratio of the energy spectrum of noise and the power spectrum of the undegraded image should be large in order to optimize the value of \( k \).

Below are the power spectra of Wiener Filter and Inverse Filter respectively:
3.2 Disadvantages:
The following are some assumptions which we assume while calculating the Wiener filtered image which are not completely correct under normal circumstances:
1. Noise and Images must be uncorrelated.
2. Noise or image has a mean value of zero.
3. The Gray levels are linear functions of the degraded image in the calculation.

3.3 Performance Parameters:
3.3.1 Peak to Noise Ratio (PSNR)
The ratio of maximum possible signal strength and noise distortion power is referred to as PSNR, affecting the efficiency of its representation.
3.3.2 Mean Square Error

\[ e = \text{Mean Square Error}; \quad E(\cdot) = \text{Expected Value} \]

Between the encoded and original image, the MSE is the cumulative square error. Where the initial image is \( f \), and the uncompressed image is \( g \). Thus, for effective compression, MSE should be as minimal as possible.

Conclusion
Over the past few decades, the topic of image improvement and reconstruction has attracted many scientists and researchers for a substantial amount of time. The Wiener filter can be considered as one of the most fundamental approaches to noise reduction among the various current techniques. It is commonly established
that, by deforming the image signal, the Wiener filter achieves noise reduction. In this paper, with the Wiener filter, we examined the inherent relation between noise reduction and image distortion. We explored the various forms of noise models and later the degradation theory from the fundamentals of image and noise estimation. Light is thrown on the degradation model, Blur model, and noise model. A comparative study is done between the Probability Mass Function and the Mean and Standard Deviations of the Gaussian, Rayleigh, Exponential, Salt and Pepper and the Uniform Noise Models. The various Restoration techniques are also discussed in detail in the paper: from the designing of Mean filters to the very popular Wiener filters a proper comparative study is done. Inverse filters had a great role in the development of wiener filters thereby leading to filters that are mostly used these days.

References
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